



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2011

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. ONE diagram sheet for answering QUESTION 12.2 is attached at the end of this question paper. Write your centre number and examination number on this sheet in the spaces provided and insert the page inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write neatly and legibly.

QUESTION 11.1 Solve for x :

1.1.1 $x(x+1) = 6$ (3)

1.1.2 $3x^2 - 4x = 8$ (4)

1.1.3 $4x^2 + 1 \geq 5x$ (4)

1.2 Consider the equation: $x^2 + 5xy + 6y^2 = 0$

1.2.1 Calculate the values of the ratio $\frac{x}{y}$. (3)

1.2.2 Hence, calculate the values of x and y if $x + y = 8$. (5)
[19]

QUESTION 22.1 Given the sequence: $4 ; x ; 32$ Determine the value(s) of x if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $P = \sum_{k=1}^{13} 3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d , the sum to n terms can be expressed as $S_n = \frac{n}{2}(2a + (n-1)d)$. (4)
[13]

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3 ; 3 ; 9 ; 6 ; 15 ; 12 ; \dots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52} - T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)
[9]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to -6 and a fifth term equal to -14 .

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)

[7]

QUESTION 5

5.1 Consider the function: $f(x) = \frac{-6}{x-3} - 1$

5.1.1 Calculate the coordinates of the y -intercept of f . (2)

5.1.2 Calculate the coordinates of the x -intercept of f . (3)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

5.1.4 For which values of x is $f(x) > 0$? (2)

5.1.5 Calculate the average gradient of f between $x = -2$ and $x = 0$. (4)

5.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0$, $b < 0$, $c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution. (4)

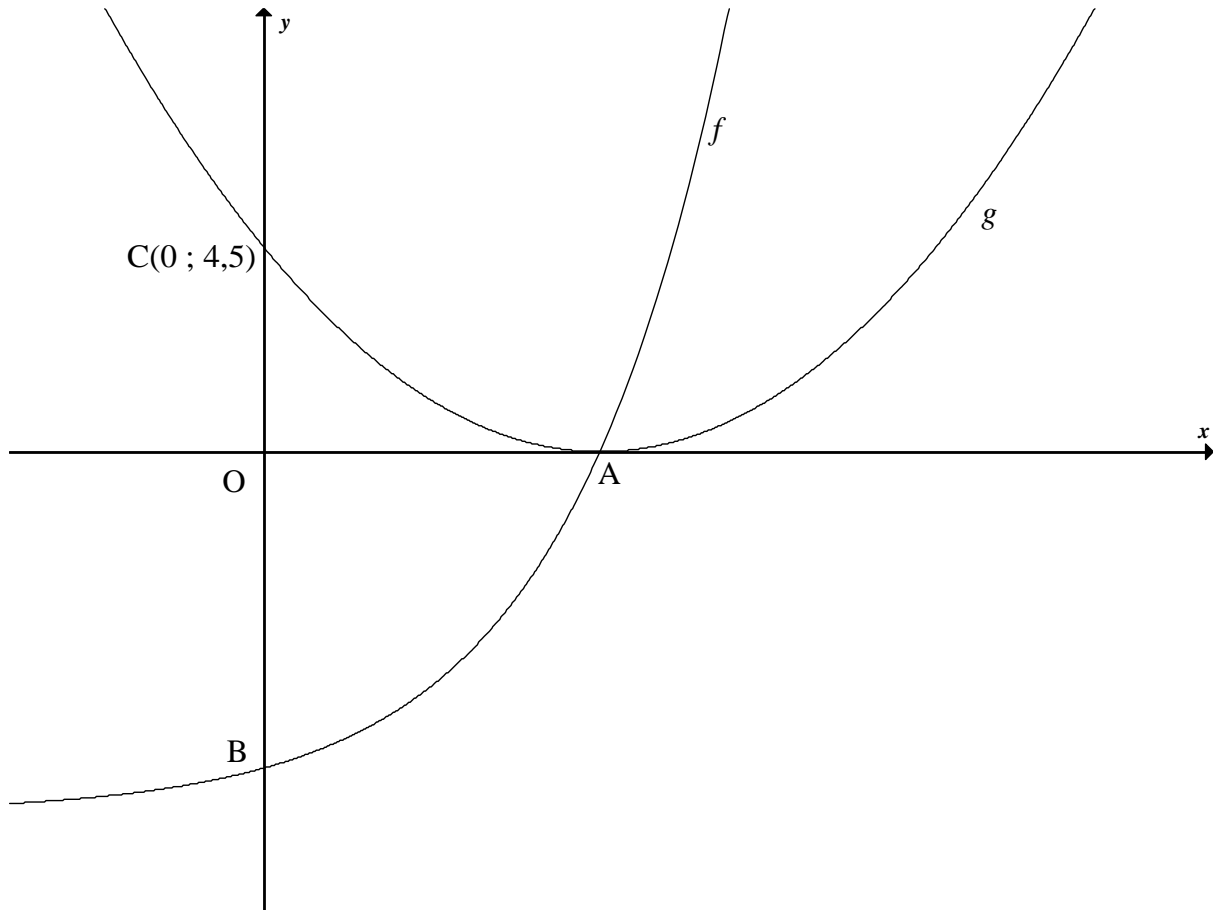
[19]

QUESTION 6

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.

B and C(0 ; 4,5) are the y-intercepts of the graphs of f and g respectively.

The two graphs intersect at A, which is the turning point of the graph of g and the x-intercept of the graphs of f and g .



- 6.1 Determine the coordinates of A and B. (4)
- 6.2 Write down an equation of the asymptote of the graph of f . (1)
- 6.3 Determine an equation of h if $h(x) = f(2x) + 8$. (2)
- 6.4 Determine an equation of h^{-1} in the form $y = \dots$ (2)
- 6.5 Write down an equation of p , if p is the reflection of h^{-1} about the x -axis. (1)
- 6.6 Calculate $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$. Show ALL your working. (4)

[14]

QUESTION 7

7.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

7.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)

7.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012). (6)
[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$. (5)

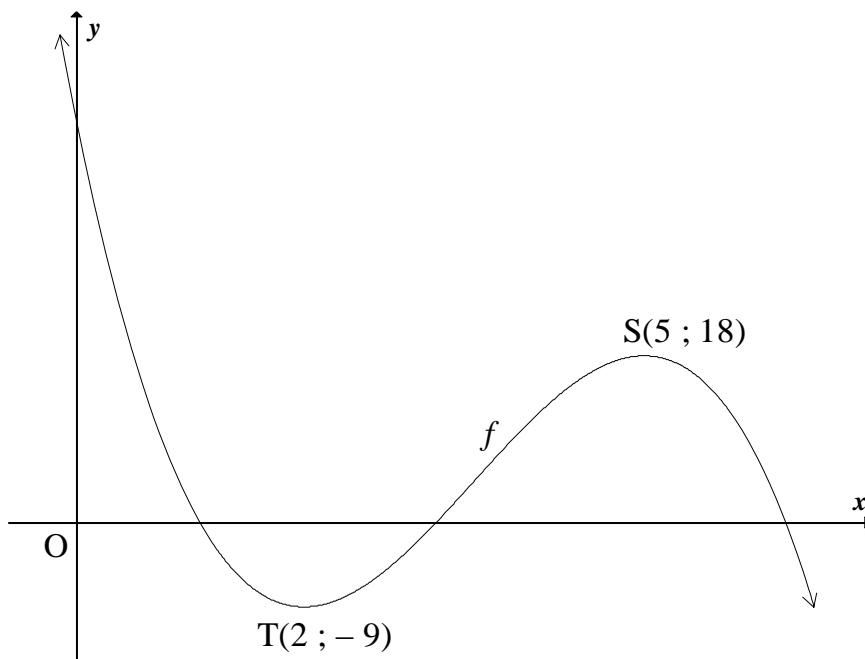
8.2 Evaluate:

8.2.1 $\frac{dy}{dx}$ if $y = \frac{3}{2x} - \frac{x^2}{2}$ (3)

8.2.2 $f'(1)$ if $f(x) = (7x+1)^2$ (4)
[12]

QUESTION 9

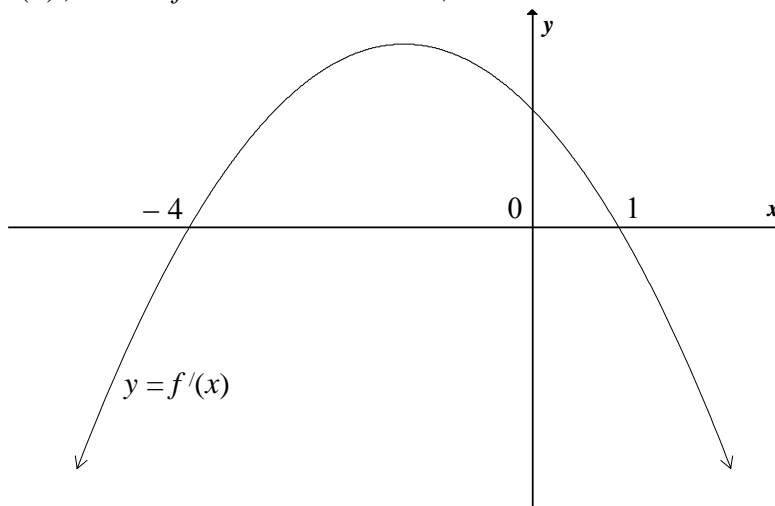
The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The turning points of the graph of f are $T(2 ; -9)$ and $S(5 ; 18)$.



- 9.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)
 - 9.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)
 - 9.3 Determine the x -value at which the graph of f has a point of inflection. (2)
- [14]**

QUESTION 10

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



Use the graph to answer the following questions:

- 10.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)
 - 10.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)
- [4]**

QUESTION 11

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

- 11.1 What is the initial volume of the water in the tank? (1)
- 11.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)
- 11.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)
- [6]**

QUESTION 12

A school is planning a trip for 500 learners. The company that will be providing the transport has two types of buses, red buses and blue buses, available.

- Each red bus has 50 seats and each blue bus has 25 seats.
- The company has at most 15 bus drivers available.
- There are at most 8 blue buses available.

Let the number of red buses hired by the school be x and the number of blue buses hired by the school be y .

- 12.1 Write down ALL the constraints, in terms of x and y , to represent the above information. (6)
- 12.2 Represent the constraints graphically on the attached DIAGRAM SHEET. Clearly indicate the feasible region. (4)
- 12.3 The cost of hiring a red bus is R600 for the day and the cost of hiring a blue bus is R300 for the day. Write down the total transport cost. (1)
- 12.4 12.4.1 Determine ALL possible values of x and y so that the cost will be a minimum. (3)
- 12.4.2 Calculate the minimum cost of hiring the buses. (2)
- 12.5 If exactly 12 bus drivers are to be used, determine the number of each type of bus which the school will now need to still ensure minimum cost. (1)
- [17]**

TOTAL: 150

CENTRE NUMBER:

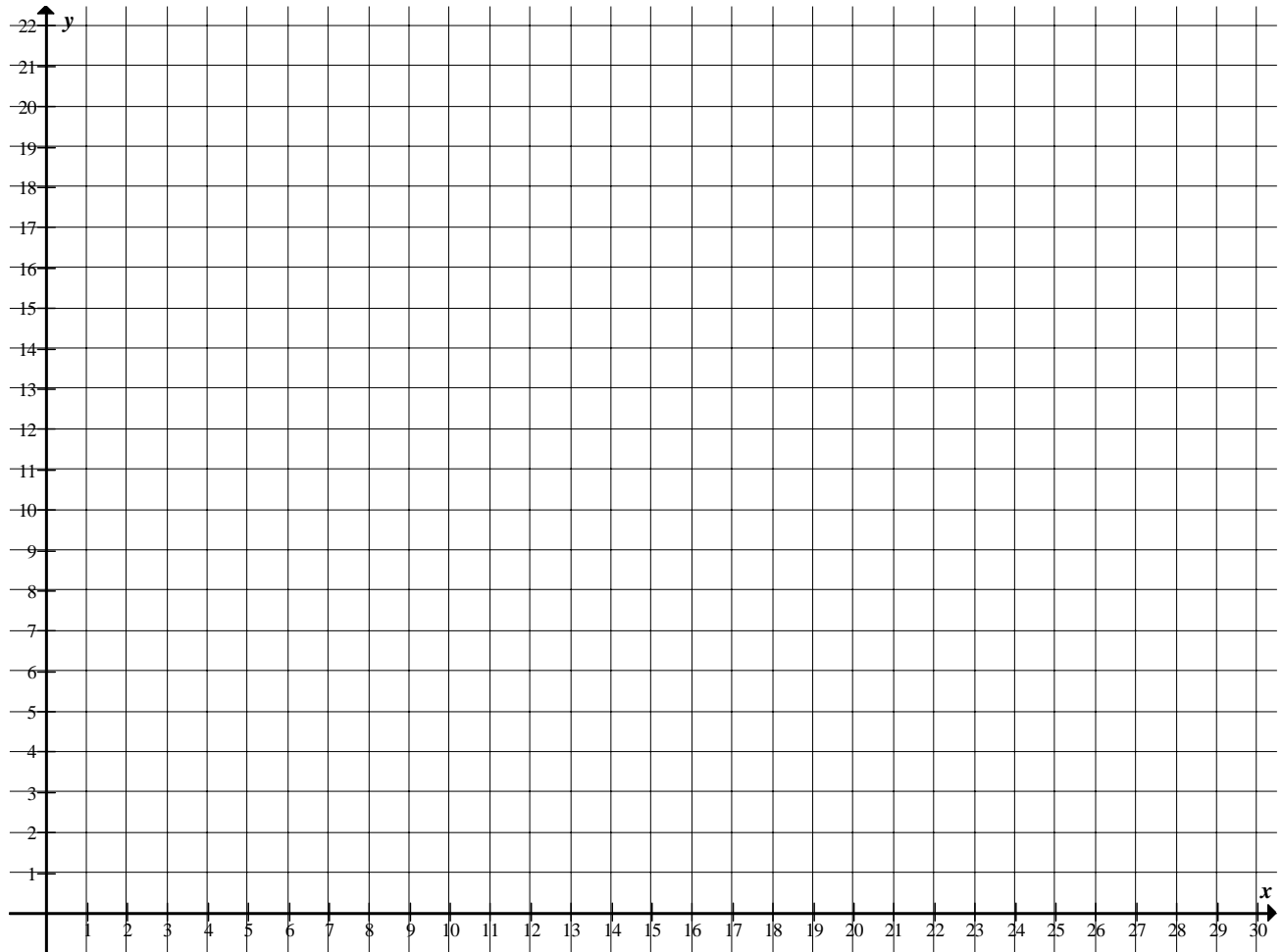
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EXAMINATION NUMBER:

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DIAGRAM SHEET 1

QUESTION 12.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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MATHEMATICS P1

NOVEMBER 2011

MEMORANDUM

MARKS: 150

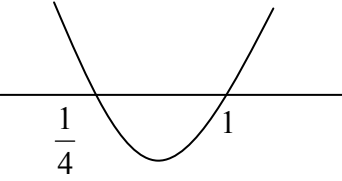
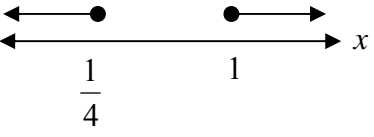
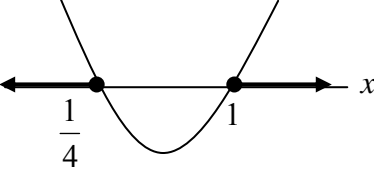
This memorandum consists of 28 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

QUESTION 1

<p>1.1.1</p>	$x(x+1) = 6$ $x^2 + x = 6$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3 \text{ or } 2$ <p>OR</p> $x^2 + x - 6 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)}$ $x = -3 \text{ or } 2$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: Answers by inspection: award 3/3 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: Answer only of $x = 2$: award 1/3 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If candidate converts equation to linear: award 0/3 marks</p> </div>	<p>✓ standard form ✓ factors ✓ answers (3)</p> <p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ answers (3)</p>
<p>1.1.2</p>	$3x^2 - 4x = 8$ $3x^2 - 4x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$ $= \frac{4 \pm \sqrt{16 + 96}}{6}$ $= \frac{4 \pm \sqrt{112}}{6}$ $= \frac{2 \pm 2\sqrt{7}}{3}$ $= 2,43 \text{ or } -1,10$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: If candidate uses incorrect formula: maximum 1/4 marks (for standard form)</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If an error in subs and gets: $\frac{4 \pm \sqrt{-80}}{6}$ and states “no solution”: maximum 3/4 marks</p> <p>If doesn't conclude with “no solution”: maximum 2/4 marks</p> </div>	<p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ $\sqrt{112}$</p> <p>✓ $\frac{4 \pm \sqrt{112}}{6}$ or decimal answer (4)</p>

	<p>OR</p> $3x^2 - 4x = 8$ $3x^2 - 4x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$ $= 2,43 \text{ or } -1,10$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers</p> </div>	<p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>																														
<p>1.1.3</p>	<p>$4x^2 + 1 \geq 5x$</p> <p>$4x^2 - 5x + 1 \geq 0$</p> <p>$(4x - 1)(x - 1) \geq 0$</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <table border="0" style="text-align: center;"> <tr> <td>+</td><td>0</td><td>-</td><td>0</td><td>+</td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black;"></td> <td></td> <td colspan="2" style="border-top: 1px solid black;"></td> </tr> <tr> <td></td><td>$\frac{1}{4}$</td><td></td><td>1</td><td></td> </tr> </table>  </div> <p>$x \leq \frac{1}{4}$ or $x \geq 1$ OR $\left(-\infty; \frac{1}{4}\right] \cup [1; \infty)$</p> <p>OR</p> <div style="display: flex; justify-content: center; align-items: center;">  </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <p>OR</p>  </div> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If candidate gives either of these correct graphical solutions but writes down the incorrect intervals or uses AND: max 3/4 marks</p> </div> <p>NOTES:</p> <p>If a candidate gives an answer of $1 \leq x \leq \frac{1}{4}$ then max 3/4 marks.</p> <p>If a candidate gives an answer of $\frac{1}{4} \leq x \leq 1$ then max 2/4 marks.</p> <p>If a candidate gives an answer of $x \leq \frac{1}{4}$ and $x \geq 1$ then max 3/4 marks.</p> <p>If the candidate leaves out the equality of the notation then penalty of 1 mark.</p> <p>If a candidate gives an answer of $x \leq \frac{1}{4}; x \geq 1$ then max 3/4 marks.</p> <p>If candidate gives $x \geq \frac{1}{4}$ and/or $x \geq 1$, BREAKDOWN: max 2/4 marks.</p> <p>If candidate gives : award 3/4 marks</p> <table border="0" style="margin-left: auto; margin-right: auto;"> <tr> <td>+</td><td>0</td><td>-</td><td>0</td><td>+</td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black;"></td> <td></td> <td colspan="2" style="border-top: 1px solid black;"></td> </tr> <tr> <td></td><td>$\frac{1}{4}$</td><td></td><td>1</td><td></td> </tr> </table>	+	0	-	0	+							$\frac{1}{4}$		1		+	0	-	0	+							$\frac{1}{4}$		1		<p>✓ factors</p> <p>✓ both critical values of $\frac{1}{4}$ and 1</p> <p>✓ or OR \cup</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
+	0	-	0	+																												
	$\frac{1}{4}$		1																													
+	0	-	0	+																												
	$\frac{1}{4}$		1																													

1.2.1	$x^2 + 5xy + 6y^2 = 0$ $(x + 3y)(x + 2y) = 0$ $x + 3y = 0 \qquad x + 2y = 0$ $x = -3y \quad \text{OR} \quad x = -2y$ $\frac{x}{y} = -3 \qquad \frac{x}{y} = -2$ <p>OR</p> <p>Let $k = \frac{x}{y}$</p> $x^2 + 5xy + 6y^2 = 0$ $\left(\frac{x}{y}\right)^2 + 5\left(\frac{x}{y}\right) + 6 = 0$ $k^2 + 5k + 6 = 0$ $(k + 3)(k + 2) = 0$ $k = -3 \quad \text{or} \quad k = -2$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p>OR</p> $x^2 + 5xy + 6y^2 = 0$ $x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$ $x = \frac{-5y \pm \sqrt{y^2}}{2}$ $x = \frac{-5y \pm y}{2}$ $x = -3y \qquad x = -2y$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p>OR</p> $x^2 + 5xy + 6y^2 = 0$ $x^2 + 5xy + \left(\frac{5}{2}y\right)^2 = -6y^2 + \left(\frac{5}{2}y\right)^2$ $\left(x + \frac{5}{2}y\right)^2 = \frac{1}{4}y^2$ $x + \frac{5}{2}y = \pm \frac{1}{2}y$ $x = -\frac{5}{2}y \pm \frac{1}{2}y$	<p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ substitutes correctly into correct formula</p> <p>✓✓ answers (3)</p> <p>✓ completing the square</p>
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	$x = -3y \quad x = -2y$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p>OR</p> <p>Let $k = \frac{x}{y}$ $x = ky$</p> $x^2 + 5xy + 6y^2 = 0$ $(ky)^2 + 5y(ky) + 6y^2 = 0$ $k^2y^2 + 5y^2k + 6y^2 = 0$ $y^2(k^2 + 5k + 6) = 0$ $(k^2 + 5k + 6) = 0$ $(k + 3)(k + 2) = 0$ $k = -3 \quad \text{or} \quad k = -2$ $\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$ <p>Note: $(x;y) = (0;0)$ is also a solution, but in this case $\frac{x}{y}$ is undefined</p> <p>OR</p> <p>Let $y = 1$,</p> $x^2 + 5x + 6 = 0$ $(x + 2)(x + 3) = 0$ $x = -2 \quad \text{or} \quad x = -3$ $\frac{x}{y} = -2 \quad \text{or} \quad \frac{x}{y} = -3$	<p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p> <p>✓ factors</p> <p>✓✓ answers (3)</p>
1.2.2	$x + y = 8 \quad x + y = 8$ $-3y + y = 8 \quad -2y + y = 8$ $-2y = 8 \quad \text{OR} \quad -y = 8$ $y = -4 \quad y = -8$ $x = 12 \quad x = 16$ <p>OR</p> $\frac{8 - y}{y} = -3 \quad \text{OR} \quad \frac{8 - y}{y} = -2$ $8 - y = -3y \quad 8 - y = -2y$ $8 = -2y \quad 8 = -y$ $y = -4 \quad y = -8$ $x = 12 \quad x = 16$	<p>✓ substitution $x = -3y$ ✓ subs $x = -2y$ ✓✓ y values ✓ both x values correct (5)</p> <p>✓ $x = 8 - y$ ✓ substitution ✓✓ y values ✓ both correct x values (5)</p>

	<p>OR</p> $x + y = 8$ $y = 8 - x$ $\frac{x}{8-x} = -3 \quad \text{OR} \quad \frac{x}{8-x} = -2$ $x = -3(8-x) \quad x = -2(8-x)$ $x = -24 + 3x \quad x = -16 + 2x$ $-2x = -24 \quad -x = -16$ $x = 12 \quad x = 16$ $y = -4 \quad y = -8$ <p>OR</p> $(x+2y)(x+3y) = 0$ $x + y = 8$ $x = 8 - y$ $(y+8)(2y+8) = 0$ $y = -8 \quad \text{or} \quad y = -4$ $x = 16 \quad x = 12$ <p>OR</p> $x = 8 - y$ $(8-y)^2 + 5(8-y)y + 6y^2 = 0$ $64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$ $2y^2 + 24y + 64 = 0$ $y^2 + 12y + 32 = 0$ $(y+8)(y+4) = 0$ $y = -8 \quad \text{or} \quad y = -4$ $x = 16 \quad x = 12$ <p>OR</p>	<p>✓ $y = 8 - x$</p> <p>✓ substitution</p> <p>✓✓ x values correct</p> <p>✓ both y values correct</p> <p>(5)</p> <p>✓ $x = 8 - y$</p> <p>✓ substitution</p> <p>✓✓ y values correct</p> <p>✓ both x values correct</p> <p>(5)</p> <p>✓ $x = 8 - y$</p> <p>✓ substitution</p> <p>✓ factors</p> <p>✓ both y values correct</p> <p>✓ both x values correct</p> <p>(5)</p>
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<p>OR $x = 8 - y$ $(8 - y)^2 + 5(8 - y)y + 6y^2 = 0$ $64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$ $2y^2 + 24y + 64 = 0$ $y^2 + 12y + 32 = 0$ $y = \frac{-12 \pm \sqrt{12^2 - 4(1)(32)}}{2(1)}$ $= \frac{-12 \pm \sqrt{16}}{2}$ $y = -8 \text{ or } y = -4$ $x = 16 \quad x = 12$</p> <p>OR $y = 8 - x$ $x^2 + 5x(8 - x) + 6(8 - x)^2 = 0$ $x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$ $2x^2 - 56x + 384 = 0$ $x^2 - 28x + 192 = 0$ $(x - 16)(x - 12) = 0$ $x = 12 \quad x = 16$ $y = -4 \text{ or } y = -8$</p> <p>OR $y = 8 - x$ $x^2 + 5x(8 - x) + 6(8 - x)^2 = 0$ $x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$ $2x^2 - 56x + 384 = 0$ $x^2 - 28x + 192 = 0$ $x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(192)}}{2(1)}$ $= \frac{28 \pm \sqrt{416}}{2}$ $x = 12 \quad x = 16$ $y = -4 \text{ or } y = -8$</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If a candidate uses the formula and replaces x for y and then answers are swapped: maximum 4/5 marks</p> </div>	<p>✓ $x = 8 - y$ ✓ substitution</p> <p>✓ substitutes into correct formula ✓ both y values correct ✓ both x values correct (5)</p> <p>✓ $y = 8 - x$ ✓ substitution</p> <p>✓ factors ✓ both x values correct ✓ both y values correct (5)</p> <p>✓ $y = 8 - x$ ✓ substitution</p> <p>✓ substitutes into correct formula ✓ both x values correct ✓ both correct y values (5) [19]</p>
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QUESTION 2

<p>2.1.1</p>	$x - 4 = 32 - x$ $2x = 36$ $x = 18$ <p>OR</p> $a = 4$ $a + 2d = 32$ $2d = 28$ $d = 14$ $x = 14 + 4$ $x = 18$ <p>OR</p> $x = \frac{4 + 32}{2} = 18$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: If answer only: award 2/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If candidate writes $x - 4 \quad 32 - x$ only (i.e. omits equality) : 0/2 marks</p> </div>	<p>✓ $T_2 - T_1 = T_3 - T_2$</p> <p>✓ answer (2)</p> <p>✓ $a + 2d = 32$ and $a = 4$</p> <p>✓ answer (2)</p> <p>✓ substitutes correctly into arithmetic mean formula i.e. $\frac{4 + 32}{2}$</p> <p>✓ answers (2)</p>
<p>2.1.2</p>	$\frac{x}{4} = \frac{32}{x}$ $x^2 = 128$ $x = \pm\sqrt{128}$ $x = \pm 8\sqrt{2} \quad \text{OR} \quad x = \pm 11,31 \quad \text{OR} \quad x = \pm 2^{\frac{7}{2}}$ <p>OR</p> $a = 4$ $r = \frac{x}{4}$ $ar^2 = 4\left(\frac{x}{4}\right)^2$ $32 = 4\left(\frac{x}{4}\right)^2$ $x^2 = 128$ $x = \pm\sqrt{128}$ $x = \pm 8\sqrt{2} \quad \text{or} \quad x = \pm 11,31 \quad \text{or} \quad x = \pm 2^{\frac{7}{2}}$ <p>OR</p> $x = \pm\sqrt{4 \times 32}$ $x = \pm\sqrt{128} \quad \text{or} \quad x = \pm 8\sqrt{2} \quad \text{or} \quad x = \pm 11,31 \quad \text{or} \quad x = \pm 2^{\frac{7}{2}}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: If candidate writes $\frac{x}{4} \quad \frac{32}{x}$ only (i.e. omits equality) : 0/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If only $x = \sqrt{128}$ then penalty 1 mark</p> </div>	<p>✓ $\frac{T_2}{T_1} = \frac{T_3}{T_2}$</p> <p>✓ $x^2 = 128$</p> <p>✓ both answers (surd or decimal or exponential form) (3)</p> <p>✓ $32 = 4\left(\frac{x}{4}\right)^2$</p> <p>✓ $x^2 = 128$</p> <p>✓ both answers (surd or decimal or exponential form) (3)</p> <p>✓✓ substitutes correctly into geometric mean formula i.e. $\pm\sqrt{4 \times 32}$</p> <p>✓ both answers (surd or decimal or exponential form) (3)</p>

2.2	$P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \dots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \dots + 3^8$ $= \frac{3^{-4}(3^{13} - 1)}{3 - 1}$ $= 9841,49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$ <p>OR</p> $P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \dots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \dots + 3^8$ $= \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \dots + 6561$ $= 9841,49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: Correct answer only: 1/4 marks only</p> </div> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If the candidate rounds off and gets 9841,46 (i.e. correct to one decimal place): DO NOT penalise for the rounding off.</p> </div>	<p>✓ $a = 3^{-4}$ or $\frac{1}{81}$</p> <p>✓ $r = 3$ ✓ subs into correct formula</p> <p>✓ answer (4)</p> <p>✓✓ expand the sum ✓ 13 terms in expansion</p> <p>✓ answer (4)</p>
2.3	$S_n = a + [a + d] + [a + 2d] + \dots + [a + (n-2)d] + [a + (n-1)d]$ $S_n = [a + (n-1)d] + [a + (n-2)d] + \dots + [a + d] + a$ $2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] + [2a + (n-1)d]$ $= n[2a + (n-1)d]$ $S_n = \frac{n}{2}[2a + (n-1)d]$ <p>OR</p> $S_n = a + [a + d] + [a + 2d] + \dots + (T_n - d) + T_n$ $S_n = T_n + (T_n - d) + \dots + [a + d] + a$ $2S_n = a + T_n + a + T_n + a + T_n + \dots + a + T_n$ $= n[a + a + (n-1)d]$ $= [2a + (n-1)d]$ $S_n = \frac{n}{2}[2a + (n-1)d]$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If a candidate uses a circular argument (eg $S_{n+1} = S_n + T_n$): max 1/4 marks (for writing out S_n)</p> </div> <p>Note: If a candidate uses a specific linear sequence, then NO marks.</p>	<p>✓ writing out S_n ✓ “reversing” S_n</p> <p>✓ expressing $2S_n$ ✓ grouping to get $2S_n = n[2a + (n-1)d]$ (4)</p> <p>✓ writing out S_n ✓ “reversing” S_n</p> <p>✓ expressing $2S_n$ ✓ grouping to get $2S_n = n[a + a + (n-1)d]$ (4)</p> <p style="text-align: right;">[13]</p>

QUESTION 3

3.1	21; 24	<p>Note: If candidate writes $T_8 = 21$ $T_7 = 24$: award 1/2 marks</p>	<p>✓ 21 ✓ 24 (2)</p>
3.2	<p>$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$</p> <p>$T_{2k-1} = 3 + 6(k-1) = 6k - 3$ and so $T_{51} = 6(26) - 3 = 153$</p> <p>$T_{52} - T_{51} = 100663296 - 153$ $= 100663143$</p> <p>OR</p> <p>Consider sequence P: 3 ; 6 ; 12 ... $P_n = 3.2^{n-1}$ $P_{26} = 3.2^{26-1} = 100663296$</p> <p>Consider sequence Q: 3 ; 9 ; 15 ... $Q_n = 6n - 3$ $Q_{26} = 6(26) - 3 = 153$ $T_{52} - T_{51} = P_{26} - Q_{26}$ $= 100663296 - 153$ $= 100663143$</p>	<p>Note: If candidate writes out all 52 terms and gets correct answer: award 5/5 marks</p> <p>Note: If candidate used $k = 52$: max 2/5</p> <p>Note: if candidate interchanges order i.e. does $T_{51} - T_{52}$: max 4/5 marks</p> <p>Note: writes out all 52 terms and subtracts $T_{51} - T_{52}$: max 4/5 marks</p>	<p>✓ 3.2^{k-1} ✓ T_{52} ✓ $6k - 3$ ✓ T_{51} ✓ answer (5)</p> <p>✓ $P_n = 3.2^{n-1}$ ✓ P_{26} ✓ $Q_n = 6n - 3$ ✓ Q_{26} ✓ answer (5)</p>

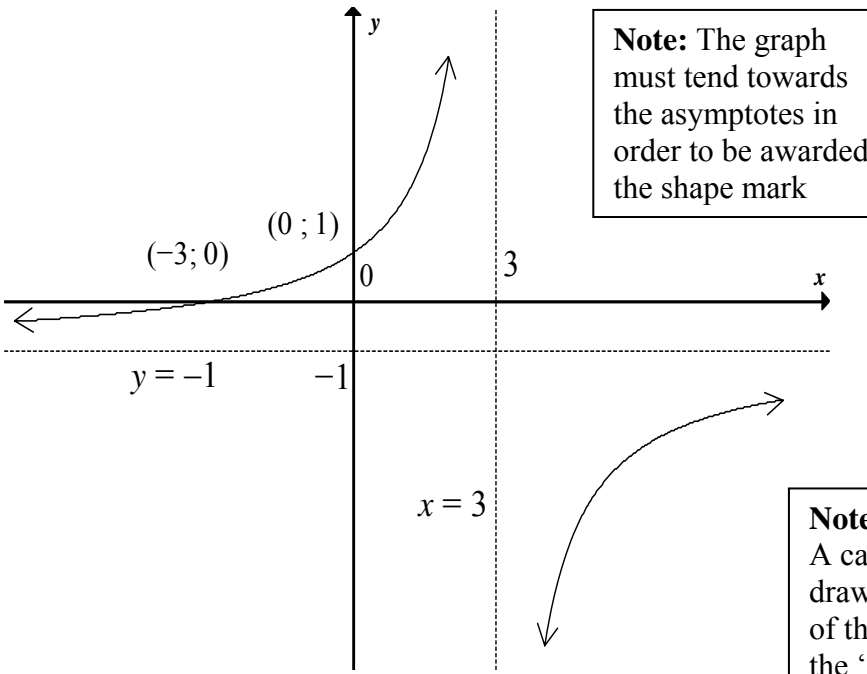
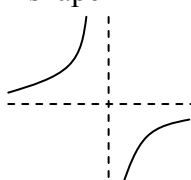
3.3	<p>For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k - 1$ for some $k \in \mathbf{N}$</p> <p>If $n = 2k$:</p> $T_n = T_{2k} = 3 \cdot 2^{k-1}$ <p>If $n = 2k - 1$:</p> $T_n = T_{2k-1}$ $= 6k - 3$ $= 3(2k - 1)$ <p>In either case, T_n has a factor of 3, so is divisible by 3.</p> <p>OR</p> $P_n = 3 \cdot 2^{n-1}$ <p>Which is a multiple of 3</p> $Q_n = 6n - 3$ $= 3(2n - 1)$ <p>Which is also a multiple of 3</p> <p>Since $T_n = Q_{2k-1}$ or $T_n = P_{2k}$ for all $n \in \mathbf{N}$, T_n is always divisible by 3</p> <p>OR</p> <p>The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3.</p>	<p>✓ factors $3 \cdot 2^{k-1}$</p> <p>✓ factors $3(2k - 1)$</p> <p>(2)</p> <p>✓ factors $3 \cdot 2^{n-1}$</p> <p>✓ factors $3(2n - 1)$</p> <p>(2)</p> <p>✓ odd multiples of 3</p> <p>✓ 3 times a power of 2</p> <p>(2)</p> <p>[9]</p>
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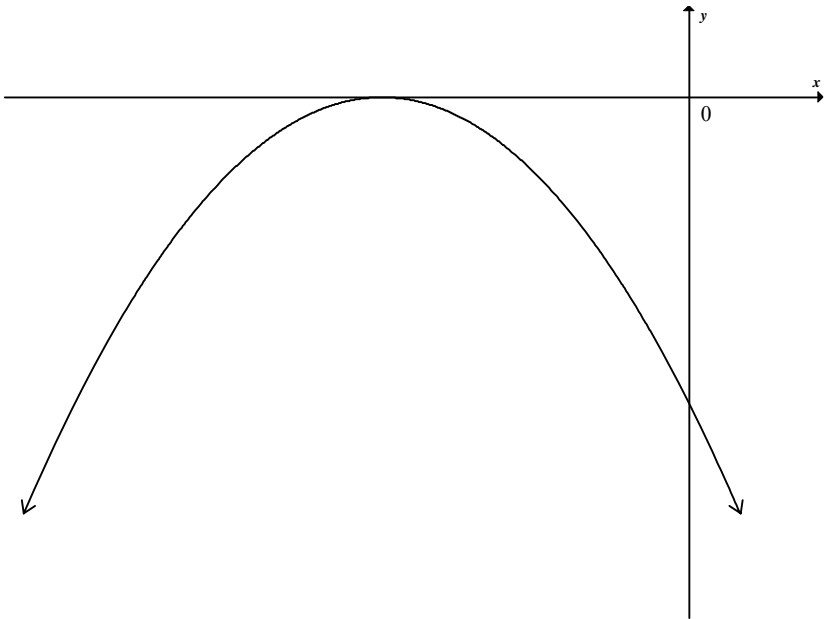
QUESTION 4

<p>4.1</p>	<p>The second, third, fourth and fifth terms are 1 ; - 6 ; T_4 and - 14</p> <p>First differences are: - 7 ; $T_4 + 6$; $- 14 - T_4$ So $T_4 + 6 + 7 = - 14 - 2T_4 - 6$ $T_4 = - 11$ $d = - 11 + 6 + 7 = 2$ or $- 14 + 22 - 6 = 2$</p> <p>OR</p> <div style="text-align: center;"> </div> <p>$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ $- 15 = (- 7 + 2d) + (- 7 + d) + - 7$ $- 15 = - 21 + 3d$ $6 = 3d$ $d = 2$</p> <p>OR</p> <p>$4a + 2b + c = 1$ $9a + 3b + c = - 6$ $5a + b = - 7$</p> <p>$25a + 5b + c = - 14$ $16a + 2b = - 8$ $10a + 2b = - 14$ $6a = 6$ $a = 1$ $d = 2a = 2$</p> <p>OR</p> <div style="text-align: center;"> </div> <p>$T_4 + 13 = - 20 - 2T_4$ $3T_4 = - 33$ $T_4 = - 11$ $d = - 11 + 13$ $d = 2$</p>	<p>✓ - 7 ✓ $T_4 + 6$ ✓ $- 14 - T_4$</p> <p>✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ ✓ answer (5)</p> <p>✓ - 7 ✓ $- 7 + d$ ✓ $- 7 + 2d$</p> <p>✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ ✓ answer (5)</p> <p>✓ $4a + 2b + c = 1$ ✓ $9a + 3b + c = - 6$</p> <p>✓ $25a + 5b + c = - 14$</p> <p>✓ solved simultaneously</p> <p>✓ answer (5)</p> <p>✓ - 7 ✓ $T_4 + 6$ ✓ $- 14 - T_4$</p> <p>✓ setting up equation ✓ answer (5)</p>
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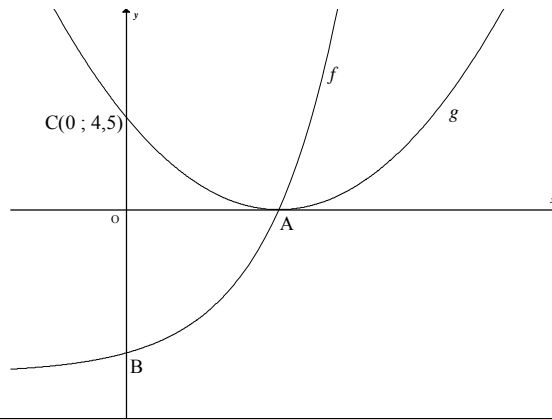
	<p>OR</p> $ \begin{array}{ccccccccc} T_1 & & T_2 & & T_3 & & T_4 & & T_5 \\ x & & 1 & & -6 & & y & & -14 \\ & \searrow & / & \searrow & / & \searrow & / & \searrow & / \\ & 1-x & & -7 & & y+6 & & -14-y & \\ & & \searrow & / & \searrow & / & & & \\ & & -8+x & & y+13 & & -20-2y & & \end{array} $ <p> $y + 13 = -20 - 2y$ $3y = -33$ $y = -11$ </p> <p>Second difference = $y + 13 = -11 + 13 = 2$</p>	<p> $\checkmark -7$ $\checkmark y + 6$ $\checkmark -14 - y$ </p> <p> \checkmark setting up equation \checkmark answer </p> <p>(5)</p>
<p>4.2</p>	$ \begin{array}{ccc} T_1 & & 1 & & -6 \\ & \searrow & / & \searrow & / \\ & -9 & & -7 & \\ & & \searrow & / & \\ & & 2 & & \end{array} $ <p> $T_1 = 10$ </p> <p>OR</p> <p> $a = 1$ $5a + b = -7$ $5(1) + b = -7$ $b = -12$ $a + b + c = 1$ $4(1) + 2(-12) + c = 1$ $c = 21$ $T_n = n^2 - 12n + 21$ $T_1 = (1)^2 - 12(1) + 21$ $= 10$ </p> <p>OR</p> <p> $T_4 + 13 = -8 + T_1$ $y + 13 = -8 + x$ $-11 + 13 = -8 + T_1$ OR $-11 + 13 = -8 + x$ $T_1 = 10$ $x = 10$ </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Answer only: award 2/2 marks</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If incorrect d in 4.1, 2/2 CA marks for $T_1 = d + 8$ (since $1 - T_1 = -7 - d$)</p> </div>	<p> \checkmark method $\checkmark T_1 = 10$ </p> <p>(2)</p> <p> \checkmark method $\checkmark T_1 = 10$ </p> <p>(2)</p> <p> \checkmark method $\checkmark T_1 = 10$ </p> <p>(2)</p> <p>[7]</p>

QUESTION 5

<p>5.1.1</p>	<p>$y = f(0)$ $= \frac{-6}{0-3} - 1$ $= 1$ $(0 ; 1)$ OR $x = 0$ and $y = 1$</p>	<p>Note: Mark 5.1.1 and 5.1.2 as a single question. If the intercepts are interchanged: max 3/5 marks</p>	<p>✓ $y = 1$ ✓ $x = 0$ (2)</p>
<p>5.1.2</p>	<p>$0 = \frac{-6}{x-3} - 1$ $1 = \frac{-6}{x-3}$ $x - 3 = -6$ $x = -3$ $(-3 ; 0)$</p>		<p>✓ $y = 0$ ✓ $x - 3 = -6$ ✓ answer (3)</p>
<p>5.1.3</p>	 <p>Note: The graph must tend towards the asymptotes in order to be awarded the shape mark</p>	<p>✓ shape</p>  <p>✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote (4)</p> <p>Note: A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks</p>	
<p>5.1.4</p>	<p>$-3 < x < 3$ OR $(-3; 3)$ OR $-3 < x$ and $x < 3$</p> <p>Note: if candidate writes $-3 < x$ only: 1/2 marks</p>	<p>Note: if candidate writes $x < 3$ only: 1/2 marks</p>	<p>✓ -3 and 3 ✓ inequality OR interval notation (2)</p>

5.1.5	$y = \frac{-6}{-2-3} - 1$ $= \frac{1}{5}$ $m = \frac{1 - \frac{1}{5}}{0 - (-2)}$ $= \frac{2}{5}$ <p>OR</p> $m = \frac{f(0) - f(-2)}{0 - (-2)}$ $= \frac{1 - \frac{1}{5}}{0 + 2}$ $= \frac{2}{5}$	<p>✓ $\frac{1}{5}$</p> <p>✓ formula ✓ substitution ✓ answer</p> <p>(4)</p> <p>✓ formula</p> <p>✓ $f(-2) = \frac{1}{5}$</p> <p>✓ substitution ✓ answer</p> <p>(4)</p>
5.2	$x = -\frac{b}{2a} < 0 \text{ since } b < 0 \text{ and } a < 0$ 	<p>✓ y-intercept negative</p> <p>✓ turning point on the x axis</p> <p>✓ turning point on the left of the y axis</p> <p>✓ maximum TP and quadratic shape</p> <p>(4) [19]</p>

QUESTION 6



6.1	$0 = 2^x - 8$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$ $A(3 ; 0)$	$f(0) = 2^0 - 8$ $= 1 - 8$ $= -7$ $B(0 ; -7)$	✓ $y = 0$ ✓ answer for A ✓ $x = 0$ ✓ answer for B (4)
6.2	$y = -8$ OR $y + 8 = 0$	Note: no CA marks	✓ answer (1)
6.3	$h(x) = f(2x) + 8$ $= (2^{2x} - 8) + 8$ $= 4^x \text{ or } 2^{2x}$	Note: answer only: award 2/2 marks	✓ $(2^{2x} - 8)$ ✓ answer of $h(x) = 4^x$ or 2^{2x} (2)
6.4	$x = 4^y$ OR $x = 2^{2y}$ $y = \log_4 x$ $2y = \log_2 x$ $y = \frac{1}{2} \log_2 x$ OR $y = \log_2 \sqrt{x}$ OR $y = \frac{\log x}{\log 4}$	Note: answer only award 2/2 marks Note: candidate works out f^{-1} and gets $y = \log_2(x + 8)$ award 1/2 marks	✓ switch x and y ✓ answer in the form $y = \dots$ (2)
6.5	$p(x) = -\log_4 x$ OR $p(x) = \log_{\frac{1}{4}} x$ OR $p(x) = \log_4 \frac{1}{x}$ OR $p(x) = -\frac{1}{2} \log_2 x$ OR $y = -\log_2 \sqrt{x}$		✓ answer (1)

<p>6.6</p> $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ <p>$x = 3$ is the axis of symmetry of g \therefore by symmetry $g(2) = g(4)$ and $g(1) = g(5)$</p> <p>Answer = $g(0) + g(3)$ $= 4,5 + 0$ $= 4,5$</p> <p>OR</p> $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $\sum_{k=0}^3 g(k) = g(0) + g(1) + g(2) + g(3)$ $\sum_{k=4}^5 g(k) = g(4) + g(5)$ <p>$x = 3$ is the axis of symmetry of g \therefore by symmetry</p> $g(4) = g(2)$ $g(5) = g(1)$ $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $= g(0) + g(3)$ $= 4,5 + 0$ $= 4,5$ <p>OR</p> $g(x) = a(x-3)^2 + 0$ $4,5 = a(0-3)^2 + 0$ $4,5 = 9a$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x-3)^2$ $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ $\sum_{k=0}^3 g(k) = g(0) + g(1) + g(2) + g(3)$ $= 4,5 + 2 + 0,5 + 0$ $= 7$	<p>✓ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)</p> <p>✓ expansion</p> <p>✓ $g(2) = g(4)$ and $g(1) = g(5)$</p> <p>✓ $g(0) + g(3)$ ✓ answer (4)</p> <p>✓ $g(x) = \frac{1}{2}(x-3)^2$</p> <p>✓ expansion</p>
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$$\begin{aligned}\sum_{k=4}^5 g(k) &= g(4) + g(5) \\ &= 0,5 + 2 \\ &= 2,5\end{aligned}$$

$$\begin{aligned}\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) \\ &= 7 - 2,5 \\ &= 4,5\end{aligned}$$

OR

$$g(x) = ax^2 + bx + c$$

$$g(k) = ak^2 + bk + c$$

$$g(0) = c$$

$$g(1) = a + b + c$$

$$g(2) = 4a + 2b + c$$

$$g(3) = 9a + 3b + c$$

$$\sum_{k=0}^3 g(k) = 14a + 6b + 4c$$

$$g(4) = 16a + 4b + c$$

$$g(5) = 25a + 9b + c$$

$$\sum_{k=4}^5 g(k) = 41a + 9b + 2c$$

$$\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) = -27a - 3b + 2c$$

$$g(x) = a(x-3)^2 + 0$$

$$4,5 = a(0-3)^2 + 0$$

$$4,5 = 9a$$

$$a = \frac{1}{2}$$

$$g(x) = \frac{1}{2}(x-3)^2$$

$$= \frac{1}{2}x^2 - 3x + \frac{9}{2}$$

$$\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k) = -27a - 3b + 2c$$

$$= -27\left(\frac{1}{2}\right) - 3(-3) + 2\left(\frac{9}{2}\right)$$

$$= 4,5$$

✓ 7 - 2,5

✓ answer

(4)

✓✓ -27a - 3b + 2c

✓ $g(x) = \frac{1}{2}(x-3)^2$

✓ answer

(4)

[14]

QUESTION 7

7.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log \frac{1}{2} = n \log 0,93$ $n = \frac{\log \frac{1}{2}}{\log 0,93}$ $= 9,55 \text{ years}$ <p style="text-align: center;">OR</p> $A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log 0,93 \frac{1}{2} = n$ $n = 9,55 \text{ years}$	$\checkmark A = \frac{P}{2}$ \checkmark subs into correct formula \checkmark log \checkmark answer <p style="text-align: right;">(4)</p>
	<p>Note: If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks</p>	<p>Note: If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$</p>

<p>7.2</p>	<p>Radesh:</p> $A = P(1 + in)$ $= 6\,000(1 + 0,085 \times 5)$ $= 8\,550$ <p style="text-align: center;">OR</p> $A = 6\,000 + 8,5\% \text{ of } 6000 \times 5$ $= 6000 + 510 \times 5$ $= 6000 + 2550$ $= 8\,550$ <p>Bonus = $0,05 \times 6\,000$</p> $= 300$ <p>Received = $8\,550 + 300$</p> $= R8\,850$ <p>Thandi:</p> $A = P(1 + i)^n$ $= 6\,000 \left(1 + \frac{0,08}{4}\right)^{20}$ $= R8\,915,68$ <p>Thandi's investment is bigger.</p>	<p>✓ 8 550</p> <p>✓ R8 850</p> <p>✓ $n = 20$</p> <p>✓ $i = \frac{0,08}{4}$</p> <p>✓ answer</p> <p>✓ choice made</p> <p style="text-align: right;">(6)</p>
<p>7.3</p>	<p>F_v = initial deposit with interest + annuity</p> $= 1\,000 \left(1 + \frac{0,15}{12}\right)^{18} + 700 \left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}} \right)$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$ <p>OR</p> <p>F_v = initial deposit with interest + annuity</p> $= 1\,000 \left(1 + \frac{0,15}{12}\right)^{18} + 700 \left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}} \right) \left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 11\,220,68 \left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$	<p>✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125</p> <p>✓ $n = 18$</p> <p>✓ $n = 18$</p> <p>✓ $1\,000 \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ $700 \left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}} \right)$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p> <p>✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125</p> <p>✓ $n = 18$</p> <p>✓ $n = 18$</p> <p>✓ $1\,000 \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ $700 \left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}} \right) \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p>

	<p>OR</p> $F_v = 300\left(1 + \frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{19} - 1}{\frac{0,15}{12}}\right)$ $= 375,17 + 14\,907,74$ $= \text{R}15\,282,91$	<ul style="list-style-type: none"> ✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 19$ (corresponding to 700) ✓ $n = 18$ (corresponding to 300) ✓ $300\left(1 + \frac{0,15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1 + \frac{0,15}{12}\right)^{19} - 1}{\frac{0,15}{12}}\right)$ ✓ answer <p style="text-align: right;">(6) [16]</p>
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QUESTION 8

8.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4(x+h)^2 - (-4x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4(x^2 + 2xh + h^2) + 4x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-4x^2 - 8xh - 4h^2 + 4x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $= \lim_{h \rightarrow 0} (-8x - 4h)$ $= -8x$ <p style="margin-top: 20px;">OR</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: Incorrect notation: no lim written: penalty 2 marks</p> <p>lim written before equals sign: penalty 1 mark</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: A candidate who gives $-8x$ only: 0/5 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: A candidate who omits brackets in the line $\lim_{h \rightarrow 0} (-8x - 4h)$: NO penalty</p> </div>	<ul style="list-style-type: none"> ✓ formula ✓ substitution ✓ expansion ✓ $-8x - 4h$ ✓ answer <p style="text-align: right;">(5)</p>
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	$f(x) = -4x^2$ $f(x+h) = -4(x+h)^2$ $= -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -8xh - 4h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $= \lim_{h \rightarrow 0} (-8x - 4h)$ $= -8x$	<ul style="list-style-type: none"> ✓ substitution ✓ expansion ✓ formula ✓ $-8x - 4h$ ✓ answer <p style="text-align: right;">(5)</p>
8.2.1	$y = \frac{3}{2x} - \frac{x^2}{2}$ $= \frac{3}{2}x^{-1} - \frac{1}{2}x^2$ $\frac{dy}{dx} = -\frac{3}{2}x^{-2} - x$ $= -\frac{3}{2x^2} - x$	<ul style="list-style-type: none"> ✓ $\frac{3}{2}x^{-1}$ ✓ $-\frac{3}{2}x^{-2}$ ✓ $-x$ <p style="text-align: right;">(3)</p>
8.2.2	$f(x) = (7x+1)^2$ $= 49x^2 + 14x + 1$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$ <p>OR</p> $f(x) = (7x+1)^2$ $f'(x) = 2(7x+1)(7) \quad \text{By the chain rule}$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p>Note: Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark</p> </div> <ul style="list-style-type: none"> ✓ multiplication ✓ $98x$ ✓ 14 ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓✓ chain rule ✓✓ answer <p style="text-align: right;">(4) [12]</p>

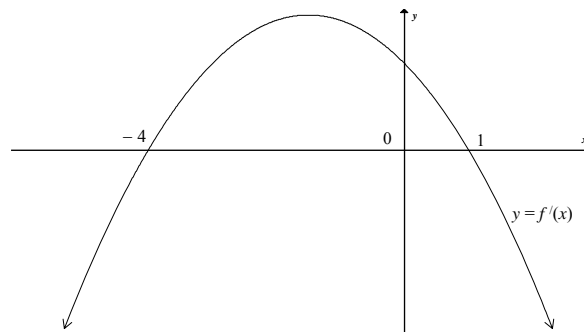
QUESTION 9

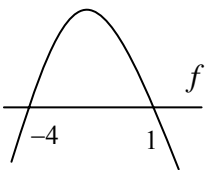
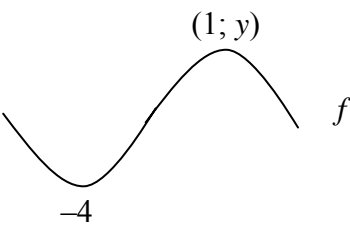

<p>9.1</p>	$f(x) = -2x^3 + ax^2 + bx + c$ $f'(x) = -6x^2 + 2ax + b$ $= -6(x-5)(x-2)$ $= -6(x^2 - 7x + 10)$ $= -6x^2 + 42x - 60$ $2a = 42$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2; -9)$ and $S(5; 18)$ lie on the curve: award max 2/7 marks</p> </div>	$\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark \checkmark -6(x-5)(x-2)$ $\checkmark b = -60$ $\checkmark 2a = 42$ $\checkmark \text{subs } (5; 18) \text{ or } (2; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>
<p>OR</p>	$a = 21; b = -60; c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: A candidate who substitutes the values of a, b and c into the function i.e. gets $f(x) = -2x^3 - 21x^2 - 60x + 43$ and then shows by substitution that $T(2; -9)$ and $S(5; 18)$ are on the curve and works out the derivative i.e. gets $f'(x) = -6x^2 - 42x - 60$ and shows (by substitution into the derivative) that the turning points are at $x = 2$ and $x = 5$ (assuming what s/he sets out to prove and proving what is given): award max 4/7 marks as follows: $\checkmark x = 2$ from $f'(x) = 0$ OR subs $x = 2$ into the derivative and gets 0 $\checkmark x = 5$ from $f'(x) = 0$ OR subs $x = 5$ into the derivative and gets 0 \checkmark substitution of $x = 2$ in f and gets -9 \checkmark substitution of $x = 5$ in f and gets 18</p> </div>	
	$f'(x) = -6x^2 + 2ax + b$ $f'(2) = -6(2)^2 + 2a(2) + b$ $0 = -24 + 4a + b$ $b = 24 - 4a$ $f'(5) = -6(5)^2 + 2a(5) + b$ $0 = -150 + 10a + b$ $0 = -150 + 10a + (24 - 4a)$ $0 = -126 + 6a$ $6a = 126$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$ $a = 21; b = -60; c = 43$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: If derivative equal to zero is not written: penalize once only</p> </div>	$\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark f'(2) = 0$ $\checkmark f'(5) = 0$ $\checkmark 6a = 126$ $\checkmark b = -60$ $\checkmark \text{subs } (5; 18) \text{ or } (2; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>

	<p>OR</p> $f(2) = -9 \text{ i.e. } -16 + 4a + 2b + c = -9$ $4a + 2b + c = 7$ $f(5) = 18 \text{ i.e. } -250 + 25a + 5b + c = 18$ $25a + 5b + c = 268$ $21a + 3b = 261$ $f'(x) = -6x^2 + 2ax + b \text{ and } f'(2) = 0 \quad \text{OR} \quad f'(5) = 0$ $4a + b = 24 \quad 10a + b = 150$ $12a + 3b = 72 \quad 30a + 3b = 450$ $9a = 189 \quad 9a = 189$ $a = \frac{189}{9} \quad \text{OR} \quad a = \frac{189}{9}$ $a = 21 \quad a = 21$ $12(21) + 3b = 72$ $3b = -180$ $b = -60$ $4a + 2b + c = 7 \quad 25a + 5b + c = 268$ $4(21) + 2(-60) + c = 7 \quad \text{OR} \quad 25(21) + 5(-60) + c = 268$ $c = 43 \quad c = 43$	$\checkmark -16 + 4a + 2b + c = -9$ $\text{and } -250 + 25a + 5b + c = 18$ $\checkmark f'(x) = -6x^2 + 2ax + b$ $\checkmark f'(2) = 0 \text{ or } f'(5) = 0$ $\checkmark 9a = 189$ $\checkmark b = -60$ $\checkmark \text{subs } (5 ; 18) \text{ or } (2 ; -9)$ $\checkmark c = 43$ <p style="text-align: right;">(7)</p>
9.2	$f'(x) = -6x^2 + 42x - 60$ $m_{\text{tan}} = -6(1)^2 + 42(1) - 60$ $= -24$ $f(1) = -2(1)^3 + 21(1)^2 - 60(1) + 43$ $= 2$ <p>Point of contact is (1 ; 2)</p> $y - 2 = -24(x - 1) \quad \text{OR} \quad y = -24x + c$ $y = -24x + 26 \quad c = 26$ $y = -24x + 26$	$\checkmark f'(x) = -6x^2 + 42x - 60$ $\checkmark \text{subs } f'(1)$ $\checkmark m_{\text{tan}} = -24$ $\checkmark f(1) = 2$ $\checkmark y - 2 = -24(x - 1)$ $\text{OR } y = -24x + 26$ <p style="text-align: right;">(5)</p>
9.3	$f'(x) = -6x^2 + 42x - 60$ $f''(x) = -12x + 42$ $0 = -12x + 42$ $x = \frac{7}{2}$ <p>OR</p>	$\checkmark f''(x) = -12x + 42$ $\checkmark x = \frac{7}{2}$ $\checkmark x = \frac{2+5}{2}$ <p style="text-align: right;">(2)</p>

$x = \frac{2+5}{2}$ $x = \frac{7}{2}$ <p>OR</p> $x = \frac{-21}{3(-2)}$ $= \frac{7}{2}$	$\checkmark x = \frac{7}{2}$ <p style="text-align: right;">(2)</p> $\checkmark x = \frac{-21}{3(-2)}$ $\checkmark x = \frac{7}{2}$ <p style="text-align: right;">(2)</p> <p style="text-align: right;">[14]</p>
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QUESTION 10



10.1	<p>x-value of turning point:</p> $x = \frac{-4+1}{2}$ $= -\frac{3}{2}$ <p>$\therefore x > -\frac{3}{2}$ OR $\therefore x \in \left(-\frac{3}{2}; \infty\right)$</p>	$\checkmark x > -\frac{3}{2}$ OR $\left(-\frac{3}{2}; \infty\right)$ <p style="text-align: right;">(1)</p>
10.2	<p>f has a local minimum at $x = -4$ because:</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>OR</p> <p>$f'(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$. $f'(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$.</p> <p>i.e.  $\therefore f$ has a local minimum at $x = -4$</p> <p>OR</p>	$\checkmark x = -4$ $\checkmark \checkmark \text{ graph}$ <p style="text-align: right;">(3)</p> $\checkmark x = -4$ $\checkmark f'(x) < 0 \text{ for } x < -4$ $\checkmark f'(x) > 0 \text{ for } -4 < x < 1$ <p style="text-align: right;">(3)</p>

	<p>OR Gradient of f changes from negative to positive at $x = -4$</p> <p>OR $f'(-4) = 0$ $f''(-4) > 0$ so graph is concave up at $x = -4$, so f has a local minimum at $x = -4$.</p>	<p>✓ $x = -4$ ✓ gradient negative for $x < -4$ ✓ gradient positive for $-4 < x < 1$ (3)</p> <p>✓ $f'(-4) = 0$ ✓ $f''(-4) > 0$ ✓ $x = -4$ (3) [4]</p>
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QUESTION 11

11.1	$V(0) = 100 - 4(0)$ $= 100$ litres	<p>✓ answer (1)</p>
11.2	<p>Rate in – rate out $= 5 - k$ l / min</p> <p>$V'(t) = -4$ l / min</p>	<p>✓ $5 - k$ ✓ -4 ✓ units stated once (3)</p>
11.3	<p>$5 - k = -4$ $k = 9$ l / min</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Answer only: award 2/2 marks</p> </div> <p>OR</p> <p>Volume at any time $t =$ initial volume + incoming total – outgoing total $100 + 5t - kt = 100 - 4t$ $5t - kt = -4t$ $9t - kt = 0$ $t(9 - k) = 0$</p> <p>At 1 minute from start, $t = 1$, $9 - k = 0$, so $k = 9$</p> <p>OR</p> <p>Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$.</p>	<p>✓ $5 - k = -4$ ✓ $k = 9$ (2)</p> <p>✓ $100 + 5t - kt = 100 - 4t$ ✓ $k = 9$ (2)</p> <p>✓✓ $k = 9$ (2) [6]</p>

QUESTION 12

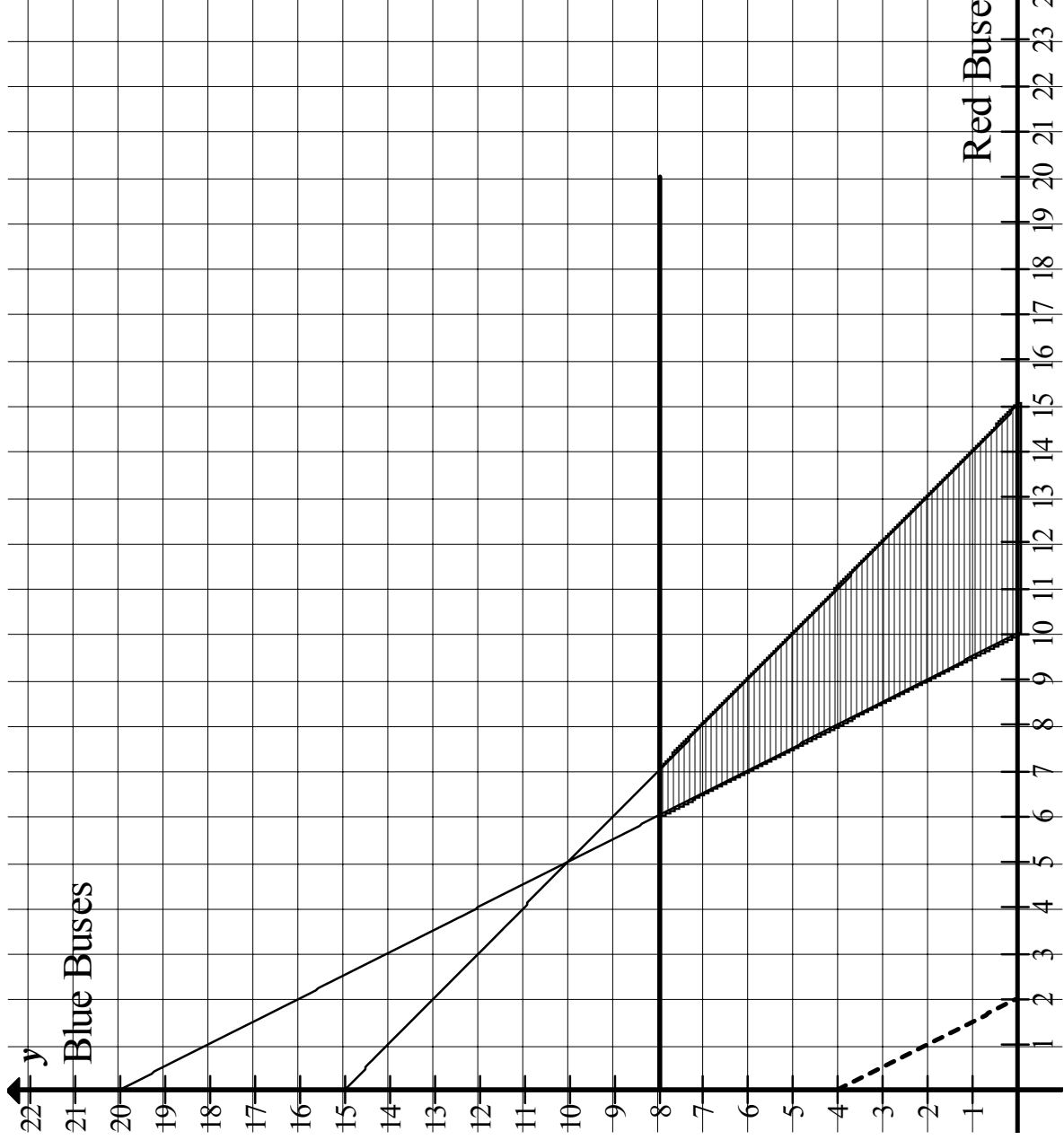
Note: If the wrong inequality $50x + 25y \leq 500$ is used, candidate wrongly says that there are more learners than available seats. Maximum of 10 marks.

12.1	$x, y \in \mathbf{N}$ $x + y \leq 15$ $50x + 25y \geq 500$ $y \leq 8$	$y \leq -x + 15$ $y \geq -2x + 20$ $y \leq 8$
<p>Note: If candidate gives $50x + 25y = 500$: max 5/6 marks</p>		$\checkmark\checkmark x + y \leq 15$ $\checkmark\checkmark y \leq 8$ $\checkmark\checkmark 50x + 25y \geq 500$
<p>Note: for the inequality's marks to be awarded, the LHS and the RHS must be correct</p>		
12.2		$\checkmark x + y \leq 15$ $\checkmark 50x + 25y \geq 500$ $\checkmark y \leq 8$ \checkmark feasible region
12.3	$C = 600x + 300y$	\checkmark answer
12.4.1	$(6 ; 8) ; (7 ; 6) ; (8 ; 4) ; (9 ; 2)$ and $(10 ; 0)$ NOTE: The gradient of the search line is $m = -\frac{2}{1}$	3 marks for all correct solutions 2 marks if only 3 or 4 correct solutions 1 mark if only 1 or 2 correct solutions
12.4.2	$C = 6(600) + 8(300) = \text{R}6\,000$ or $C = 7(600) + 6(300) = \text{R}6\,000$ or $C = 8(600) + 4(300) = \text{R}6\,000$ or $C = 9(600) + 2(300) = \text{R}6\,000$ or $C = 10(600) + 0(300) = \text{R}6\,000$	\checkmark subs \checkmark answer
12.5	8 red ; 4 blue	\checkmark answer

[17]

TOTAL: 150

QUESTION 12.2





basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2011

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. ONE diagram sheet for QUESTION 4.2 is attached at the end of this question paper. Write your centre number and examination number on this sheet in the spaces provided and insert the sheet inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of this question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write neatly and legibly.

QUESTION 1

Fifteen members of a basketball team took part in a tournament. Each player was allowed the same amount of time on the court. The points scored by each player at the end of the tournament are shown below.

27	28	30	32	34	38	41	42	43	43	44	46	53	56	62
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- 1.1 Determine the median of the given data. (1)
- 1.2 Determine the interquartile range for the data. (3)
- 1.3 Draw a box and whisker diagram to represent the data. (3)
- 1.4 Use the box and whisker diagram to comment on the points scored by the players in this team. (2)
- [9]**

QUESTION 2

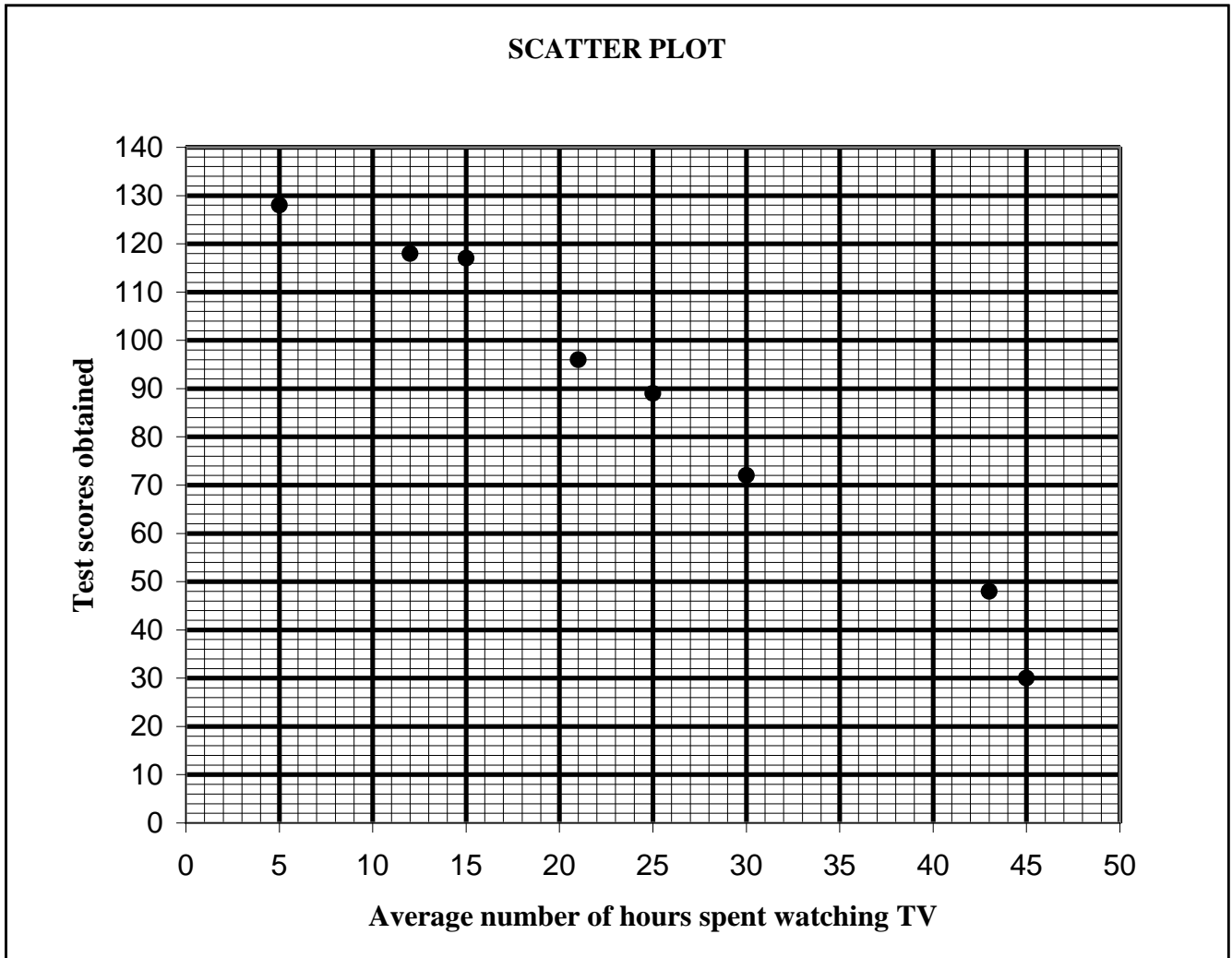
The scores for 8 golfers who played a single round of golf on the same golf course are shown below.

69	71	75	74	78	70	72	71
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- 2.1 Calculate the mean score. (2)
- 2.2 Calculate the standard deviation for the data. (2)
- 2.3 How many golfers' scores lie outside one standard deviation of the mean? (2)
- [6]**

QUESTION 3

A group of 8 learners was randomly selected from a class. The performance of these learners in a standardised test (which counted 150 marks) and the average number of hours they spend watching TV each week was recorded. The data is represented in the scatter plot below.



- 3.1 What is the lowest test score for this group of learners? (1)
 - 3.2 Does the data display a linear, quadratic or exponential relationship? Justify your choice. (2)
 - 3.3 What conclusion can be reached about the learners' test scores and the average number of hours they spend watching TV? (1)
 - 3.4 Another learner from the class watches 35 hours of TV per week. Using the given information, predict his/her performance in the test. (2)
- [6]**

QUESTION 4

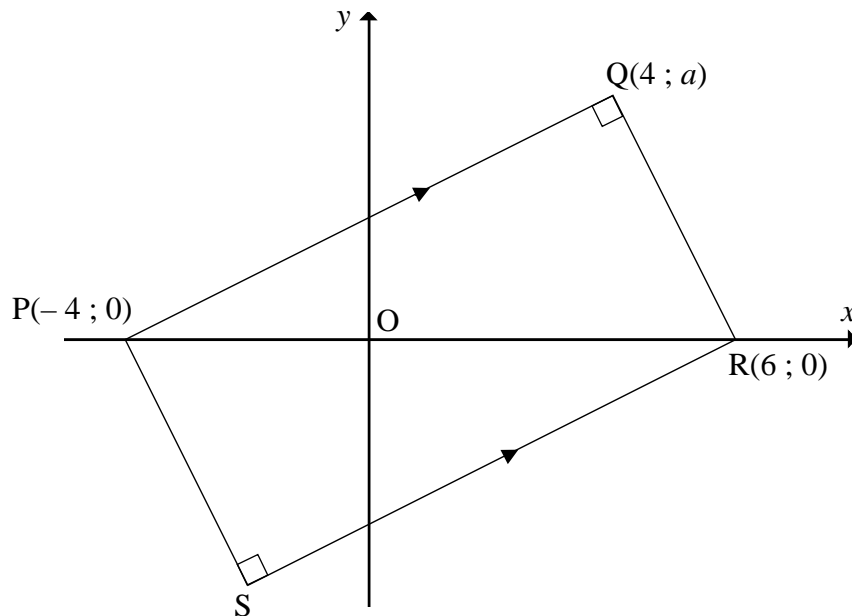
Thirty learners were asked to answer a question in Mathematics. The time taken, in minutes, to answer the question correctly, is shown in the frequency table below.

TIME, T (IN MINUTES)	NUMBER OF LEARNERS
$1 \leq t < 3$	3
$3 \leq t < 5$	6
$5 \leq t < 7$	7
$7 \leq t < 9$	8
$9 \leq t < 11$	5
$11 \leq t < 13$	1

- 4.1 Construct a cumulative frequency table for the data. (3)
- 4.2 Draw a cumulative frequency graph (ogive) of the above data on the grid provided on DIAGRAM SHEET 1. (4)
- 4.3 If a learner answers the question correctly in less than 4 minutes, then he/she is classified as a 'gifted learner'. Estimate the percentage of 'gifted learners' in this group. (2)
- [9]**

QUESTION 5

In the diagram below, PQRS is a rectangle with vertices $P(-4 ; 0)$, $Q(4 ; a)$, $R(6 ; 0)$ and S. Q lies in the first quadrant.



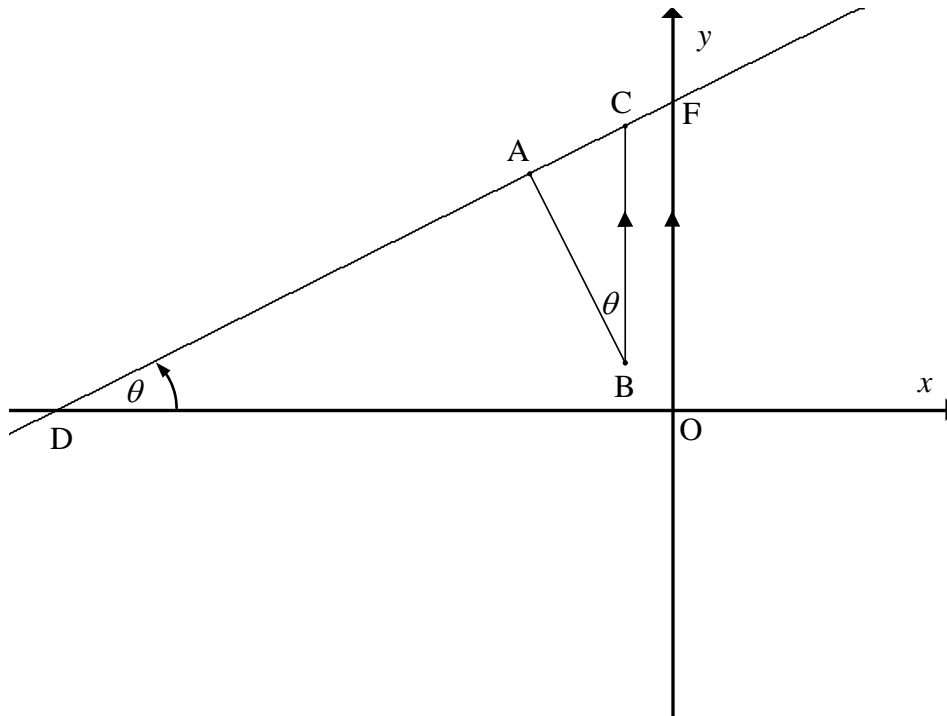
- 5.1 Show that $a = 4$. (4)
- 5.2 Determine the equation of the straight line passing through the points S and R in the form $y = mx + c$. (4)
- 5.3 Calculate the coordinates of S. (4)
- 5.4 Calculate the length of PR. (2)
- 5.5 Determine the equation of the circle that has diameter PR. Give the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- 5.6 Show that Q is a point on the circle in QUESTION 5.5. (2)
- 5.7 Rectangle PQRS undergoes the transformation $(x ; y) \rightarrow (x + k ; y + l)$ where k and l are numbers. What is the minimum value of $k + l$ so that the image of PQRS lies in the first quadrant (that is, $x \geq 0$ and $y \geq 0$)? (3)

[22]

QUESTION 6

The circle with centre $B(-1 ; 1)$ and radius $\sqrt{20}$ is shown. BC is parallel to the y -axis and $CB = 5$. The tangent to the circle at A passes through C .

$\hat{A}BC = \hat{A}DO = \theta$



- 6.1 Determine the coordinates of C . (2)
 - 6.2 Calculate the length of CA . (3)
 - 6.3 Write down the value of $\tan \theta$. (1)
 - 6.4 Show that the gradient of AB is -2 . (2)
 - 6.5 Determine the coordinates of A . (6)
 - 6.6 Calculate the ratio of the area of $\triangle ABC$ to the area of $\triangle ODF$. Simplify your answer. (5)
- [19]**

QUESTION 7

7.1 The following transformation is applied to all points:

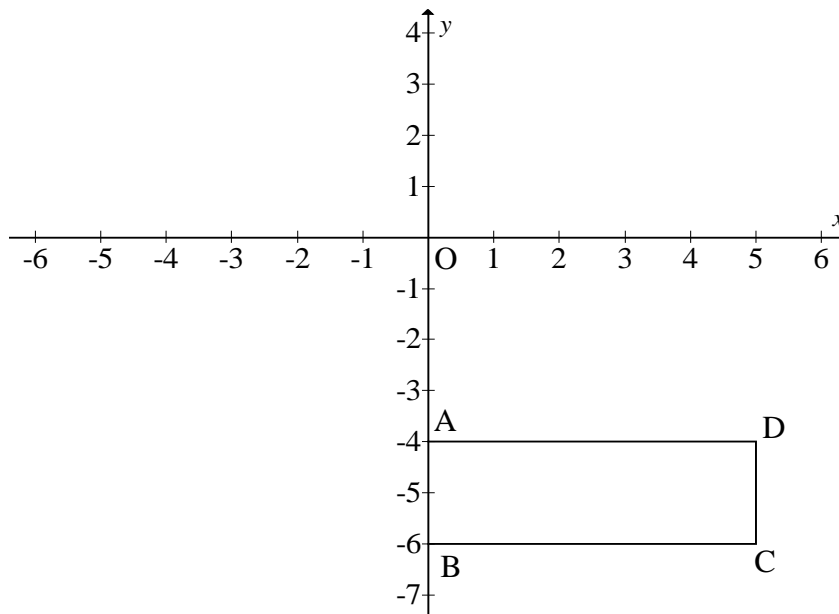
- Firstly, a point is translated by 4 units to the right.
- Then it is rotated through 180° about the origin.

Write down the general rule that represents the above transformation in the form $(x; y) \rightarrow \dots$

(4)

7.2 Reflect the circle with centre $C(-5; -2)$ and radius of 4 units about the line $y = x$.
Give the equation of the new circle in the form $ax^2 + by^2 + cx + dy + e = 0$.

(4)

[8]**QUESTION 8**

In the diagram, A is the point $(0; -4)$, $AB = 2$ and $AD = 5$. Rectangle ABCD is rotated about the origin to form rectangle $A'B'C'D'$. After the rotation the image of point A is $A'(4; 0)$.

8.1 Describe the transformation fully in words. (2)

8.2 Write down the coordinates of D' . (2)

8.3 If ABCD is reflected about the line $x = -1$ to form EFGH, write down the coordinates of G, the image of C. (2)

8.4 If ABCD is enlarged by a scale factor of $\frac{3}{2}$ through the origin to form MNPR, determine the value of $\text{area ABCD} \times \text{area MNPR}$. (3)

[9]

QUESTION 9

9.1 If $\tan A = \frac{3}{\sqrt{40}}$ and $0^\circ < \hat{A} < 90^\circ$, determine the values of the following with the aid of a sketch and without using a calculator. Leave your answers in surd form, if necessary.

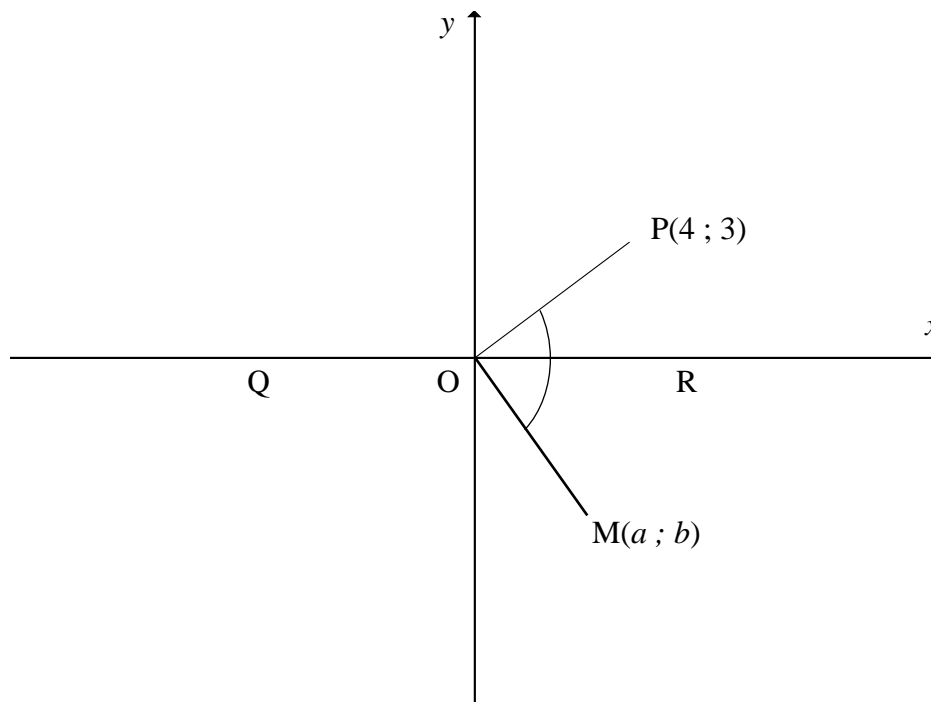
9.1.1 $\cos A$ (3)

9.1.2 $\sin (180^\circ + A)$ (2)

9.2 Without using a calculator, determine the value of the following expression:

$$\frac{\cos 100^\circ}{\sin (-10^\circ)} \times \tan^2 120^\circ$$
 (6)

9.3 P(4 ; 3) and M(a ; b) are points on a circle with the origin as centre. Q and R are x-intercepts of the circle.



9.3.1 Write down the numerical value of $\sin \hat{R}OP$. (2)

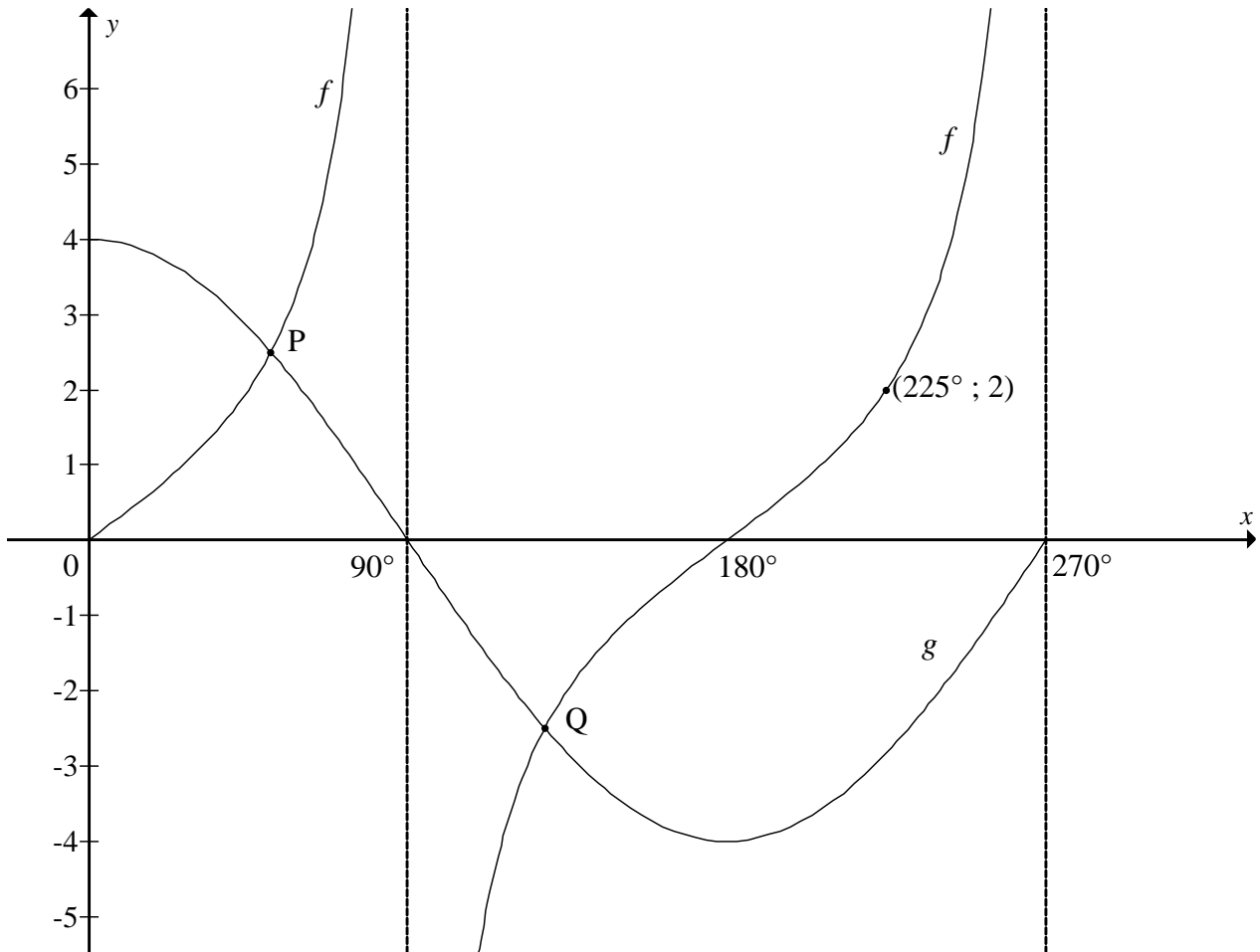
9.3.2 Calculate the size of $\hat{Q}OP$. (2)

9.3.3 If obtuse $\hat{P}OM = 115^\circ$, calculate the value of a , the x -coordinate of M, correct to TWO decimal places. (3)

[18]

QUESTION 10

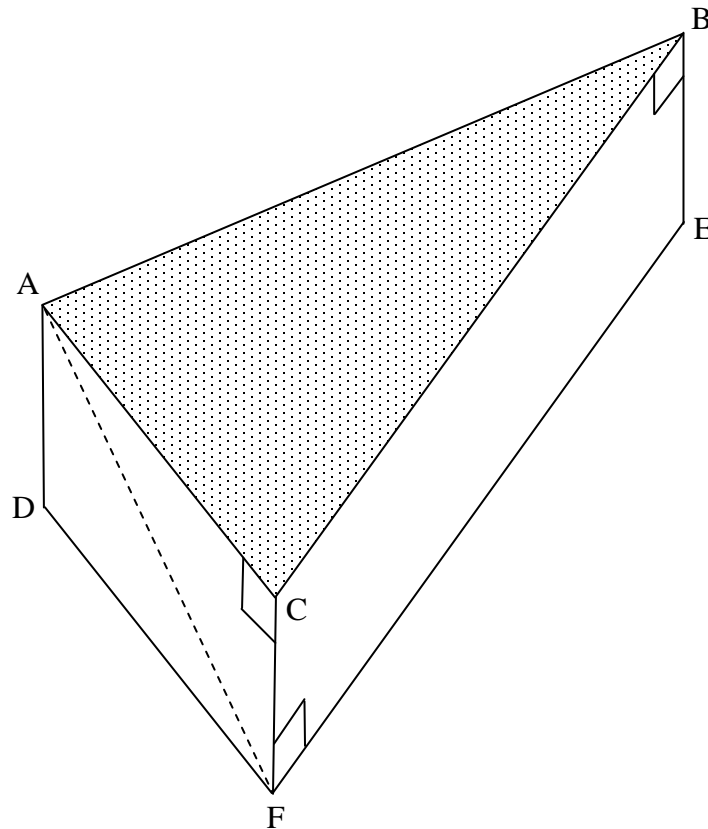
The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0^\circ \leq x \leq 270^\circ$ are shown in the diagram below. The point $(225^\circ ; 2)$ lies on f . The graphs intersect at points P and Q.



- 10.1 Determine the numerical values of a and b . (4)
 - 10.2 Determine the minimum value of $g(x) + 2$. (2)
 - 10.3 Determine the period of $f\left(\frac{1}{2}x\right)$. (2)
 - 10.4 Show that, if the x -coordinate of P is θ , then the x -coordinate of Q is $(180^\circ - \theta)$. (4)
- [12]**

QUESTION 11

The figure below represents a triangular right prism with $BA = BC = 5$ units, $\hat{ABC} = 50^\circ$ and $\hat{FAC} = 25^\circ$.



- 11.1 Determine the area of $\triangle ABC$. (2)
- 11.2 Calculate the length of AC. (3)
- 11.3 Hence, determine the height FC of the prism. (3)
- [8]**

QUESTION 1212.1 Prove that, if $\cos(\alpha - x) \neq 0$,

$$\frac{\sin(x + 450^\circ - \alpha)}{\cos(\alpha - x)} = 1. \quad (3)$$

12.2 Determine the general solution of $\cos 2x = 1 - 3\cos x$. (7)12.3 12.3.1 Prove that, for angles A and B ,

$$\frac{\sin A}{\sin B} - \frac{\cos A}{\cos B} = \frac{2\sin(A - B)}{\sin 2B} \quad (4)$$

12.3.2 Hence, or otherwise, **without using a calculator**, show that:

$$(a) \quad \frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = 4\cos 2B \quad (3)$$

$$(b) \quad \frac{1}{\sin 18^\circ} = 4\cos 36^\circ \quad (3)$$

$$(c) \quad \sin 18^\circ \text{ is a solution of the cubic equation } 8x^3 - 4x + 1 = 0 \quad (4)$$

[24]

TOTAL: 150

CENTRE NUMBER:

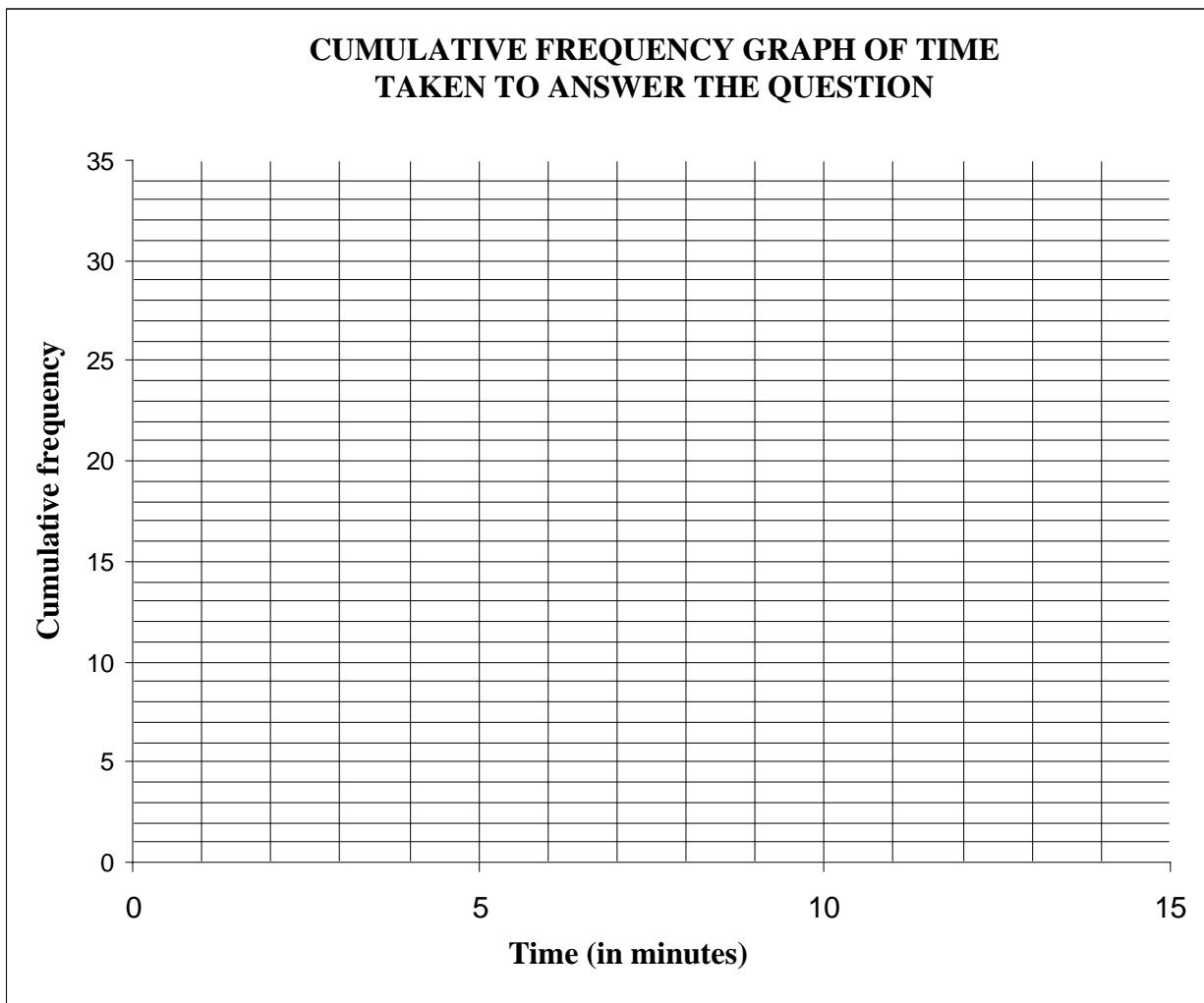
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EXAMINATION NUMBER:

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DIAGRAM SHEET 1

QUESTION 4.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2011

MEMORANDUM

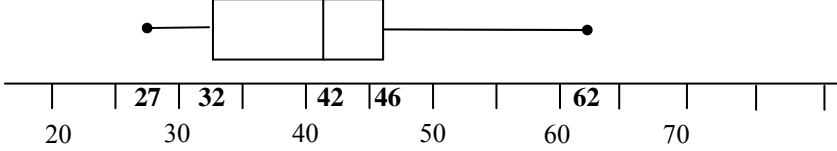
MARKS: 150

This memorandum consists of 22 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is not acceptable.

QUESTION 1

1.1	Median = 42	✓ answer (1)
1.2	Lower quartile = 32 Upper quartile = 46 Inter quartile range = $46 - 32 = 14$	✓ lower quartile ✓ upper quartile ✓ answer (3)
1.3		✓ box-and-whisker with a median ✓ skewness ✓ indicating <u>5</u> <u>number summary</u> 27; 32; 42; 46; 62 or correct scale (3)
1.4	There is a greater spread of scores to the right of the median (42). <p style="text-align: center;">OR</p> There is a greater spread of scores in the top 50%. <p style="text-align: center;">OR</p> The spread of the scores on the left hand side of the median is closer to each other. <p style="text-align: center;">OR</p> The greatest spread of scores lies between Q_3 and the maximum value. Note: <ul style="list-style-type: none"> • Description about the spread based on the box-and-whisker diagram must be accepted. • If it is indicated that it is skewed to the left because the mean is less than the median: full marks 	✓ greater spread ✓ right of median (42) (2) ✓ greater spread ✓ top 50% (2) ✓ spread closer ✓ left of median (2) ✓ greater spread ✓ between Q_3 and max (2) <p style="text-align: right;">[9]</p>

QUESTION 2

2.1	$\text{Mean} = \frac{\sum_{i=1}^n x_i}{n} = \frac{580}{8} = 72,5$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Answer only: FULL MARKS </div> <p>Note: If rounded off to 73: 1 mark</p>	✓ 580 ✓ answer (2)
2.2	Standard deviation (σ) = 2,78 (2,783882181...) Note: If rounded off to 2,8: 1 mark	✓✓ answer (2)
2.3	<p>∴ 2 golfers' scores lie outside 1 standard deviation of the mean. The interval for 1 standard deviation of the mean is (72,5 – 2,78 ; 72,5 + 2,78) = (69,72 ; 75,28)</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Answer only: FULL MARKS </div>	✓ interval ✓ number (2) [6]

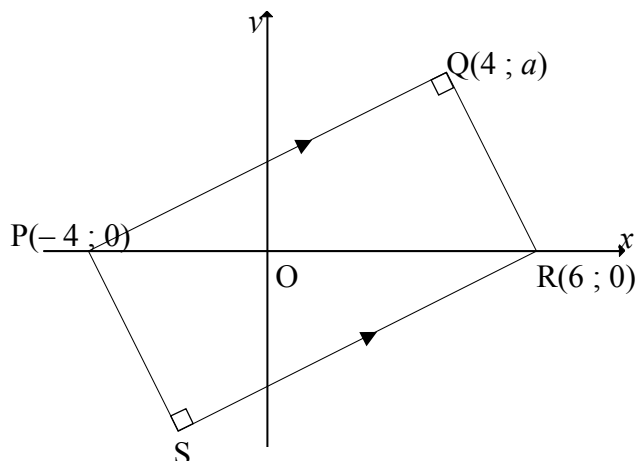
QUESTION 3

3.1	30	✓ 30 (1)
3.2	Linear, the points seem to form a straight line.	✓ linear ✓ reason (2)
3.3	The greater the number of hours spent watching TV, the lower the test scores <p style="text-align: center;">OR</p> The less time a person spends watching TV, the higher the test score. <p style="text-align: center;">OR</p> Negative correlation between the variables <p style="text-align: center;">OR</p> Indirect relationship between the variables	✓ deduction (1)
3.4	60 marks. (Accept 50 -70 marks)	✓✓ deduction (2) [6]

QUESTION 4

<p>4.1</p>	<table border="1"> <thead> <tr> <th>TIME</th> <th>FREQUENCY</th> <th>CUMULATIVE FREQUENCY</th> </tr> </thead> <tbody> <tr> <td>$1 \leq t < 3$</td> <td>3</td> <td>3</td> </tr> <tr> <td>$3 \leq t < 5$</td> <td>6</td> <td>9</td> </tr> <tr> <td>$5 \leq t < 7$</td> <td>7</td> <td>16</td> </tr> <tr> <td>$7 \leq t < 9$</td> <td>8</td> <td>24</td> </tr> <tr> <td>$9 \leq t < 11$</td> <td>5</td> <td>29</td> </tr> <tr> <td>$11 \leq t < 13$</td> <td>1</td> <td>30</td> </tr> </tbody> </table>	TIME	FREQUENCY	CUMULATIVE FREQUENCY	$1 \leq t < 3$	3	3	$3 \leq t < 5$	6	9	$5 \leq t < 7$	7	16	$7 \leq t < 9$	8	24	$9 \leq t < 11$	5	29	$11 \leq t < 13$	1	30	<p>One mark for every two correct cumulative frequency values</p> <p>(3)</p>
TIME	FREQUENCY	CUMULATIVE FREQUENCY																					
$1 \leq t < 3$	3	3																					
$3 \leq t < 5$	6	9																					
$5 \leq t < 7$	7	16																					
$7 \leq t < 9$	8	24																					
$9 \leq t < 11$	5	29																					
$11 \leq t < 13$	1	30																					
<p>Note: Only cumulative frequency column – full marks</p>																							
<p>4.2</p>	<p style="text-align: center;">Cumulative Frequency Graph of time taken to answer</p>	<p>✓ upper limit ✓ cumulative frequency (at least 4 of 6 y-values correctly plotted)</p> <p>✓ grounding at (1 ; 0)</p> <p>✓ shape (not joined by a ruler; smooth curve)</p> <p>(4)</p>																					
<p>4.3</p>	<p>Estimated number of learners that took less than 4 minutes: approximately 5 learners (Accept 6) Approximate percentage = 16,67% (Accept 20%)</p> <p>Note: If using 9 learners and approximate percentage = 30%: 1 mark If using 5,5 learners and approximate percentage = 18,33%: 1 mark</p>	<p>✓ 5 learners ✓ 16,67%</p> <p>(2) [9]</p>																					

QUESTION 5



<p>5.1</p>	$m_{PQ} \times m_{QR} = -1$ $\left(\frac{a-0}{4+4}\right)\left(\frac{a-0}{4-6}\right) = -1$ $\left(\frac{a}{8}\right)\left(\frac{a}{-2}\right) = -1$ $\frac{a^2}{-16} = -1$ $a^2 = 16$ $a = \pm 4$ $a = 4; \text{ since } a > 0$ <p style="text-align: center;">OR</p> $PQ^2 + QR^2 = PR^2$ $(8^2 + a^2) + (a^2 + 2^2) = 10^2$ $\therefore 2a^2 = 32$ $\therefore a^2 = 16$ $\therefore a = 4$ <p style="text-align: center;">OR</p> <p>Let A be the midpoint of diagonal PR.</p> <p>Then $A\left(\frac{-4+6}{2}; \frac{0+0}{2}\right) = A(1; 0)$.</p> <p>AQ = AR (diagonals equal and bisect each other)</p> $AQ^2 = AR^2$ $(1-4)^2 + (0-a)^2 = 5^2$ $9 + a^2 = 25$ $a^2 = 16$ $a = 4$ <p>Note: If candidate uses $a = 4$ at the beginning, then zero marks.</p>	$\checkmark \frac{a-0}{4+4} \text{ or } \frac{a}{8}$ $\checkmark \frac{a-0}{4-6} \text{ or } \frac{a}{-2}$ <p>✓ using gradient of perpendicular lines</p> $\checkmark a^2 = 16$ <p style="text-align: right;">(4)</p> <p>✓ using Pythagoras</p> $\checkmark (8^2 + a^2)$ $+ (a^2 + 2^2)$ $\checkmark 10^2$ $\checkmark a^2 = 16$ <p style="text-align: right;">(4)</p> <p>✓ (1; 0) is centre</p> <p>✓ AQ = AR</p> $\checkmark 3^2 + a^2 = 5^2$ $\checkmark a^2 = 16$ <p style="text-align: right;">(4)</p>
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5.2	<p>Equation of line SR:</p> $m_{PQ} = \frac{4-0}{4-(-4)} = \frac{1}{2}$ $m_{SR} = m_{PQ} = \frac{1}{2} \quad \text{PQ} \parallel \text{SR}$ $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{2}(x - 6)$ $y = \frac{1}{2}x - 3$ <p style="text-align: center;">OR</p>	<p>✓ $m_{PQ} = \frac{1}{2}$</p> <p>✓ $m_{SR} = \frac{1}{2}$</p> <p>✓ substitution of m and (6 ; 0)</p> <p>✓ standard form (4)</p>
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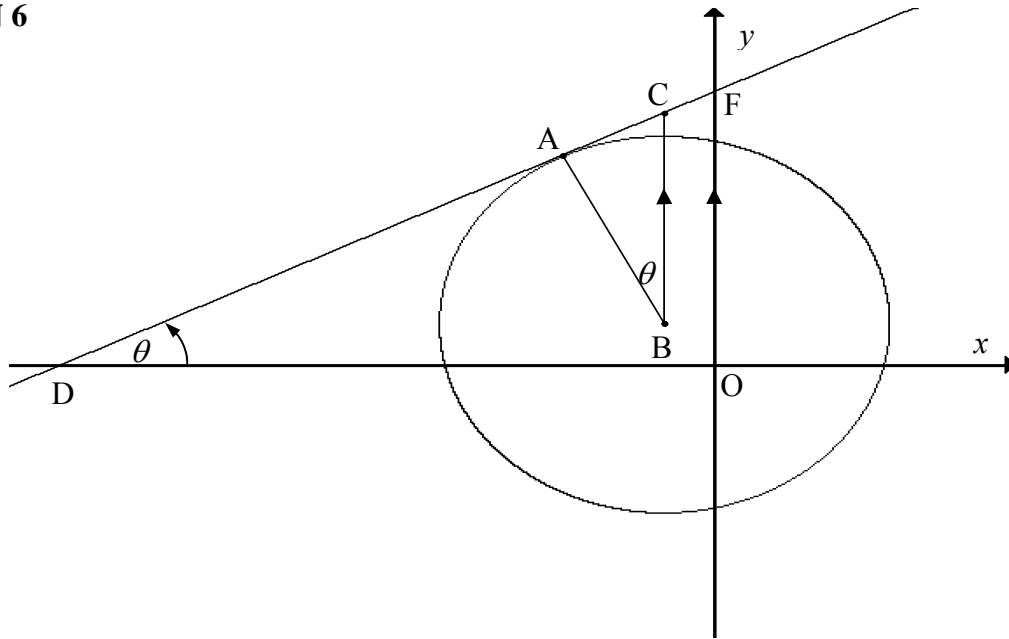
	$m_{PQ} = \frac{1}{2}$ $m_{PQ} = m_{SR} = \frac{1}{2} \quad \text{PQ} \parallel \text{SR}$ $y = \frac{1}{2}x + c$ $0 = \left(\frac{1}{2}\right)\left(\frac{6}{1}\right) + c$ $-3 = c$ $y = \frac{1}{2}x - 3$ <p style="text-align: center;">OR</p> <p>S(-2 ; -4) (translation)</p> $m_{RS} = \frac{0+4}{6+2} = \frac{1}{2}$ $\therefore y + 4 = \frac{1}{2}(x + 2)$ $\therefore y = \frac{1}{2}x - 3$	<p>✓ $m_{PQ} = \frac{1}{2}$</p> <p>✓ $m_{SR} = \frac{1}{2}$</p> <p>✓ substitution of m and (6 ; 0)</p> <p>✓ standard form</p> <p>✓ S(-2 ; -4)</p> <p>✓ $m_{SR} = \frac{1}{2}$</p> <p>✓ substitution of m and (-2 ; -4)</p> <p>✓ standard form (4)</p>
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5.3	<p>Eq. of RS: $y = \frac{1}{2}x - 3$</p> <p>Eq. of SP: $y - 0 = -2(x + 4)$</p> $\therefore \frac{1}{2}x - 3 = -2(x + 4)$ $\therefore x = -2$ $y = -4$ <p style="text-align: center;">OR</p> <div style="border: 1px solid black; width: fit-content; margin: 10px auto; padding: 5px; text-align: center;"> Answer only: FULL MARKS </div>	<p>✓ $m = -2$</p> <p>✓ eq. of SP</p> <p>✓ value of x</p> <p>✓ value of y (4)</p>
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	<p>Midpoint PR = M$\left(\frac{-4+6}{2}; \frac{0+0}{2}\right) = (1 ; 0)$</p> <p>Let S(x; y). Then since M(1 ; 0) is this, the midpoint of QS is:</p> $\frac{x_1 + x_2}{2} = 1 \qquad \frac{y_1 + y_2}{2} = 0$ $\therefore \frac{x+4}{2} = 1 \qquad \text{and} \qquad \frac{y+4}{2} = 0$ $x+4 = 2 \qquad y+4 = 0$ $x = -2 \qquad y = -4$ <p style="text-align: center;">OR</p> <p>The translation that sends Q(4 ; 4) to R(6 ; 0) also sends P(-4 ; 0) to S.</p> $(6 ; 0) = (4 + 2 ; 4 - 4)$ $\therefore S = (-4 + 2 ; 0 - 4) = (-2 ; -4)$ <p style="text-align: center;">OR</p> <p>The translation that sends Q(4 ; 4) to P(-4 ; 0) also sends R(6 ; 0) to S.</p> $(-4 ; 0) = (4 - 8 ; 4 - 4)$ $\therefore S = (6 - 8 ; 0 - 4) = (-2 ; -4)$ <p style="text-align: center;">OR</p> <p>$m_{PQ} = m_{SR}$</p> $\frac{1}{2} = \frac{y}{x-6}$ $2y = x - 6 \quad (1)$ <p>$m_{PS} = m_{SR}$</p> $\frac{y}{x+4} = \frac{4}{-2}$ $-2y = 4x + 16 \quad (2)$ <p>(1) + (2) : $0 = 5x + 10$</p> $x = -2$ <p>Substitute : $2y = -2 - 6 = -8$</p> $y = -4$	<p>$\checkmark \frac{x+4}{2} = 1$</p> <p>$\checkmark \frac{y+4}{2} = 0$</p> <p>$\checkmark$ value of x</p> <p>\checkmark value of y</p> <p style="text-align: right;">(4)</p> <p>\checkmark method</p> <p>$\checkmark 2$ or $x + 2$</p> <p>$\checkmark -4$ or $y - 4$</p> <p>\checkmark answer</p> <p style="text-align: right;">(4)</p> <p>\checkmark method</p> <p>$\checkmark -8$ or $x - 8$</p> <p>$\checkmark -4$ or $y - 4$</p> <p>\checkmark answer</p> <p style="text-align: right;">(4)</p> <p>\checkmark equations using the gradient</p> <p>\checkmark adding the equations</p> <p>\checkmark value of x</p> <p>\checkmark value of y</p> <p style="text-align: right;">(4)</p>
<p>5.4</p>	<p>$PR = 6 - (-4)$</p> <p>$= 10$</p> <p style="text-align: center;">OR</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p>Answer only: FULL MARKS</p> </div> <p>$PR^2 = (6+4)^2 + (0-0)^2$</p> <p>$PR = 10$</p>	<p>$\checkmark 6 - (-4)$</p> <p>$\checkmark 10$</p> <p style="text-align: right;">(2)</p> <p>\checkmark substitution in correct formula</p> <p>$\checkmark 10$</p> <p style="text-align: right;">(2)</p>

<p>5.5</p>	<p>midpoint $PR = \left(\frac{6+(-4)}{2}; \frac{0+0}{2} \right) = (1; 0)$</p> <p>radius of circle $= \frac{1}{2} PR = 5$ units</p> <p>$\therefore (x-1)^2 + (y-0)^2 = 5^2$ $(x-1)^2 + y^2 = 25$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Answer only: FULL MARKS</p> </div>	<p>✓ midpoint</p> <p>✓ radius</p> <p>✓ eq. of circle in correct form</p> <p style="text-align: right;">(3)</p>
<p>5.6</p>	<p>$(x-1)^2 + y^2 = 25$ substitute $Q(4; 4)$: LHS $= (4-1)^2 + 4^2$ $= 25$ $=$ RHS $\therefore Q$ is a point on the circle Note: If substitute point into equation resulting in $25 = 25$: 1 mark No conclusion: 1 mark</p> <p style="text-align: center;">OR</p> <p>Distance from centre $(1; 0)$ to $Q(4; 4)$ $\therefore Q$ is a point on circle, $r = 5$</p> <p style="text-align: center;">OR</p> <p>PR is the diameter of circle PQR therefore Q lies on circle $(\hat{PQR} = 90^\circ)$</p> <p style="text-align: center;">OR</p> <p>$(4-1)^2 + y^2 = 25$ $y^2 = 16$ $\therefore y = 4$ $\therefore Q$ is a point on the circle</p> <p style="text-align: center;">OR</p> <p>$(x-1)^2 + 4^2 = 25$ $(x-1)^2 = 9$ $x-1 = 3$ $x = 4$ $\therefore Q$ is a point on the circle</p>	<p>✓ substitute $Q(4;4)$</p> <p>✓ LHS = RHS</p> <p style="text-align: right;">(2)</p> <p>✓ = 5</p> <p>✓ conclusion</p> <p style="text-align: right;">(2)</p> <p>✓ diameter</p> <p>✓ $\hat{PQR} = 90^\circ$</p> <p style="text-align: right;">(2)</p> <p>✓ substitute $x = 4$</p> <p>✓ conclusion</p> <p style="text-align: right;">(2)</p> <p>✓ substitute $y = 4$</p> <p>✓ conclusion</p> <p style="text-align: right;">(2)</p>
<p>5.7</p>	<p>P needs to shift at least 4 units to the right and S needs to shift at least 4 units up for the image of PQRS in first quadrant. \therefore minimum value of k is 4 and minimum value of l is 4 \therefore minimum value of $k + l$ is 8</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Answer only: FULL MARKS</p> </div> <p>Note: No CA mark applies in 5.7 if k and l are not minimums.</p>	<p>✓ $k = 4$</p> <p>✓ $l = 4$</p> <p>✓ $k + l = 8$</p> <p style="text-align: right;">(3) [22]</p>

QUESTION 6



6.1	$x_C = x_B = -1$ $y_C = y_B + 5 = 6$ $\therefore C(-1 ; 6)$	✓ value of x ✓ value of y (2)
6.2	$BA \perp CA$ (tangent \perp radius) $\therefore CA^2 = BC^2 - AB^2$ (Pythagoras) $= (5)^2 - (\sqrt{20})^2 = 5$ $\therefore CA = \sqrt{5}$ or 2,24 units	✓ $BA \perp CA$ or $\hat{BAC} = 90^\circ$ ✓ substitution into Pythagoras ✓ answer (3)
6.3	$\tan \theta = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$	✓ tan ratio (in any form) (1)
6.4	$m_{DC} \times m_{AB} = -1$ $m_{DC} = \tan \theta = \frac{1}{2}$ $m_{DC} = \frac{1}{2}$ $m_{AB} = -2$	✓ $m_{DC} \times m_{AB} = -1$ ✓ $m_{DC} = \tan \theta = \frac{1}{2}$ (2)

6.5	<p>Eq. of DC: $y - 6 = \frac{1}{2}(x + 1)$ $y = \frac{1}{2}x + \frac{13}{2}$</p> <p>Eq. of AB: $y - 1 = -2(x + 1)$ $y = -2x - 1$</p> <p>$-2x - 1 = \frac{1}{2}x + \frac{13}{2}$ $-\frac{5}{2}x = \frac{15}{2}$ $x = -3$ $y = -2(-3) - 1$ $y = 5$ $\therefore A(-3 ; 5)$</p> <p style="text-align: center;">OR</p> <p>Eq. of DC: $y - 6 = \frac{1}{2}(x + 1)$ $y = \frac{1}{2}x + \frac{13}{2}$</p> <p>Eq. of AB: $y - 1 = -2(x + 1)$ $y = -2x - 1$</p> <p><u>At A:</u> $x - 2(-2x - 1) + 13 = 0$ $x + 4x + 2 + 13 = 0$ $5x = -15$ $x = -3$ and $y = -2(-3) - 1 = 5$ $\therefore A(-3 ; 5)$</p> <p style="text-align: center;">OR</p>	<div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> Answer only: (-3 ; 5): 1 mark </div> <p>✓ DC: subst m and $(-1 ; 6)$ ✓ eq. of DC</p> <p>✓ eq. of AB</p> <p>✓ equating equations</p> <p>✓ value of x ✓ value of y (6)</p> <p>✓ DC: subst m and $(-1 ; 6)$ ✓ eq. of DC</p> <p>✓ subst m and $(-1 ; 1)$ ✓ eq. of AB</p> <p>✓ value of x ✓ value of y (6)</p>
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	<p>Eq. of DC: $y - 6 = \frac{1}{2}(x + 1)$ $y = \frac{1}{2}x + \frac{13}{2}$</p> <p>Eq. of circle: $(x + 1)^2 + (y - 1)^2 = 20$</p> <p><u>At A:</u> $(x + 1)^2 + (\frac{1}{2}x + \frac{13}{2} - 1)^2 = 20$ $(x + 1)^2 + (\frac{1}{2}x + \frac{11}{2})^2 = 20$ $1\frac{1}{4}x^2 + \frac{15}{2}x + 11\frac{1}{4} = 0$ $\therefore x^2 + 6x + 9 = 0$ $(x + 3)^2 = 0$ $\therefore x = -3$ and $y = \frac{1}{2}(-3) + \frac{13}{2} = 5$ $\therefore A(-3 ; 5)$</p>	<p>✓ DC: subst m and $(-1 ; 6)$ ✓ eq. of DC</p> <p>✓ substitution</p> <p>✓ $x^2 + 6x + 9 = 0$</p> <p>✓ value of x</p> <p>✓ value of y (6)</p>
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OR

Draw $AE \perp BC$

$$\cos \theta = \frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}} = \frac{BE}{2\sqrt{5}}$$

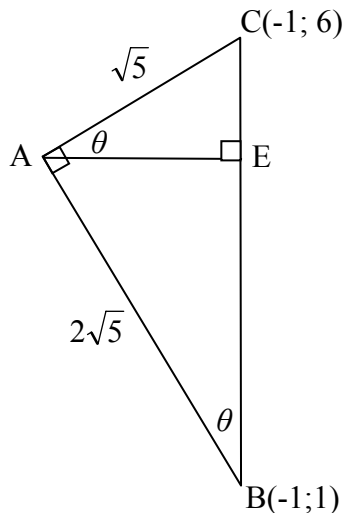
$$\therefore AE = \frac{2 \times 5}{5} = 2$$

$$BE = \frac{4 \times 5}{5} = 4$$

$$x_A = -1 - AE = -1 - 2 = -3$$

$$\therefore y_A = 1 + BE = 4 + 1 = 5$$

$$\therefore A(-3 ; 5)$$



$$\checkmark \frac{2\sqrt{5}}{5} = \frac{AE}{\sqrt{5}}$$

$$\checkmark AE = 2$$

$$\checkmark \frac{2\sqrt{5}}{5} = \frac{BE}{2\sqrt{5}}$$

$$\checkmark BE = 1$$

$$\checkmark -3$$

$$\checkmark 5$$

(6)

OR

$$(x+1)^2 + (y-1)^2 = 20 \quad (1)$$

$$y = -2x - 1 \quad (2)$$

$$(x+1)^2 + (-2x-2)^2 = 20$$

$$x^2 + 2x + 1 + 4x^2 + 8x + 4 - 20 = 0$$

$$5x^2 + 10x - 15 = 0$$

$$x^2 + 10x - 15 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3 \text{ or } x \neq 1$$

subst (1) in (2)

$$\therefore y = 5$$

✓ subst m and (-1;1)

✓ eq of AB

✓ eq of circle

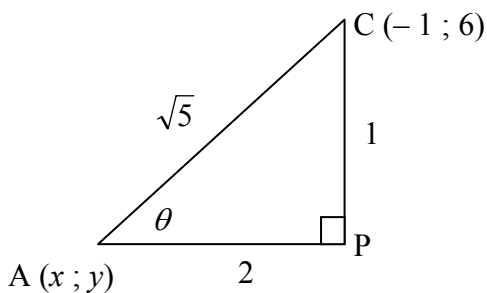
✓ substitution

✓ value of x

✓ value of y (6)

OR

Equation AC : $y = \frac{1}{2}x + 6\frac{1}{2}$



$\tan \theta = \frac{1}{2}$

$\theta = 26,57^\circ$

$AP = \sqrt{5} \cos 26,57^\circ$

$AP = 2$

$CP = \sqrt{5} \sin 26,57^\circ$

$CP = 1$

$\therefore x = -1 - 2 = -3$

$y = 6 - 1 = 5$

$\therefore A(-3; 5)$

✓ $\theta = 26,57^\circ$

✓

$AP = \sqrt{5} \cos 26,57^\circ$

✓ $AP = 2$

✓ $CP = 1$

✓ value of x

✓ value of y

(6)

6.6

Area $\Delta ABC = \frac{1}{2}(\sqrt{5})(\sqrt{20}) = 5$

Eqn. of DC is $y = \frac{1}{2}x + \frac{13}{2}$

Therefore $OF = \frac{13}{2}$ and $OD = 13$.

Area $\Delta ODF = \frac{1}{2}\left(\frac{13}{2}\right)(13) = \frac{169}{4}$

Area ΔABC : Area $\Delta ODF = 5 : \frac{169}{4} = 20 : 169$

OR

$DF^2 = 13^2 + \left(\frac{13}{2}\right)^2 = \frac{845}{4}$

$DF = \frac{13 \cdot \sqrt{5}}{2}$

$$\frac{\Delta_{ABC}}{\Delta_{ODF}} = \frac{\frac{1}{2}(5)(\sqrt{20}) \sin \theta}{\frac{1}{2}(13)\left(\frac{13 \cdot \sqrt{5}}{2}\right) \sin \theta}$$

$$= \frac{20}{169}$$

✓ $\frac{1}{2}(\sqrt{5})(\sqrt{20})$

✓ $OF = \frac{13}{2}$

✓ $OD = 13$

✓ $\frac{1}{2}\left(\frac{13}{2}\right)(13)$

✓ answer

(5)

✓ $= 13^2$

$+ \left(\frac{13}{2}\right)^2 = \frac{845}{4}$

✓ $DF = \frac{13 \cdot \sqrt{5}}{2}$

✓ $\frac{1}{2}(5)(\sqrt{20}) \sin \theta$

✓ $\frac{1}{2}(13)\left(\frac{13 \cdot \sqrt{5}}{2}\right) \sin \theta$

✓ answer (5)

	OR	
	ΔODF is an enlargement of ΔABC $\therefore \text{area } \Delta ABC : \text{area } \Delta ODF = AB^2 : OD^2 = 20 : OD^2$ Equation of DC is $y = \frac{1}{2}x + \frac{13}{2}$ $x_D = -13$ $OD = 13$ $\therefore \text{area } \Delta ABC : \text{area } \Delta ODF = AB^2 : OD^2 = 20 : 169$	✓ enlargement ✓✓ $AB^2 : OD^2 = 20 : OD^2$ ✓ - 13 ✓ answer (5)
		[19]

QUESTION 7

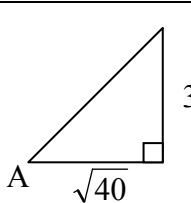
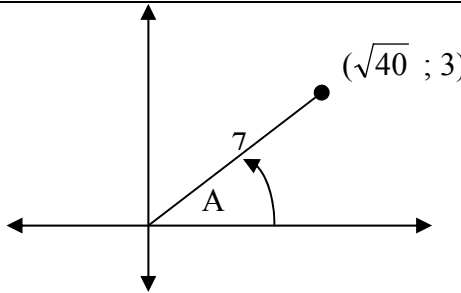
7.1	$(x; y) \rightarrow (x + 4; y) \rightarrow (-x - 4; -y)$ OR $(x; y) \rightarrow (-x - 4; -y)$	✓ $x + 4$ ✓ y ✓ $-x - 4$ ✓ $-y$ (4)
7.2	New centre = $(-2; -5)$ $(x + 2)^2 + (y + 5)^2 = 16$ $x^2 + 4x + 4 + y^2 + 10y + 25 - 16 = 0$ $x^2 + y^2 + 4x + 10y + 13 = 0$	✓ $(-2; -5)$ ✓ $(x + 2)^2 + (y + 5)^2$ ✓ 16 ✓ simplification (4)
		[8]

QUESTION 8

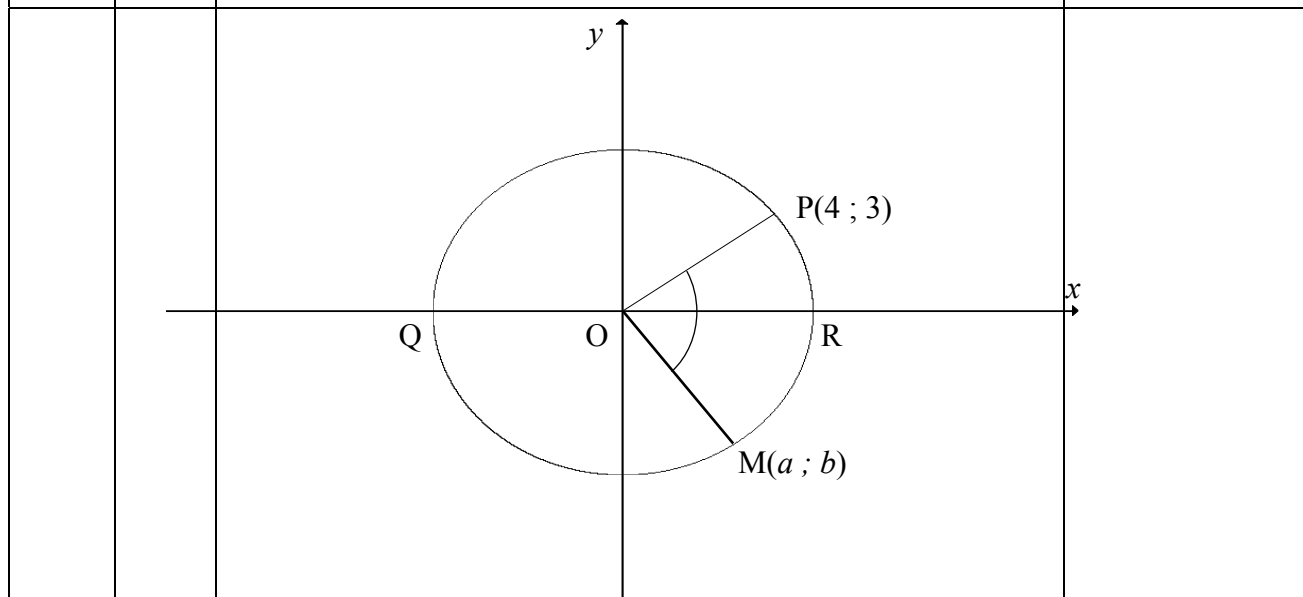
8.1	Rotation of 90° anticlockwise about the origin. OR Rotation of 270° clockwise about the origin. Note: if reflection of 90° anticlockwise: 0 marks	✓ rotation 90° ✓ anticlockwise (2) ✓ rotation 270° ✓ clockwise (2)
8.2	$D(5; -4)$ $D'(4; 5)$	✓ 4 ✓ 5 (2)
8.3	$G(-7; -6)$	✓ -7 ✓ -6 (2)
8.4	Area ABCD = $5 \times 2 = 10$ square units Area MNRP = $10 \times \left(\frac{3}{2}\right)^2 = \frac{45}{2}$ Area ABCD \times Area MNRP $= 10 \times \frac{9}{4} \times 10$ $= 225 \text{ (units)}^4$ OR	✓ area ABCD = 10 ✓ area MNRP $= \frac{45}{2}$ ✓ 225 (3)

	$\text{Product} = \left(\frac{3}{2}\right)^2 \times (\text{area ABCD})^2$ $= \frac{9}{4} \times (5 \times 2)^2$ $= 225 (\text{units})^4$ <p>Note: CA will apply if $\left(\frac{3}{2}\right)^2$ used in calculation.</p>	<p>✓ $\left(\frac{3}{2}\right)^2$</p> <p>✓ 10^2</p> <p>✓ 225</p> <p style="text-align: right;">(3) [9]</p>
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QUESTION 9

9.1	9.1.1	 <p style="text-align: center;">or</p>  <p> $r^2 = 40 + 9$ $r = 7$ $\cos A = \frac{\sqrt{40}}{7}$ </p>	<p>✓ sketch</p> <p>✓ $r = 7$</p> <p>✓ $\frac{\sqrt{40}}{7}$</p> <p style="text-align: right;">(3)</p>
	9.1.2	<p> $\sin(180^\circ + A)$ $= -\sin A$ $= -\frac{3}{7}$ </p> <p style="text-align: center;">OR</p> <p> $\sin(180^\circ + A) = \sin 180^\circ \cdot \cos A + \cos 180^\circ \cdot \sin A$ $= 0 \cdot \cos A - 1 \cdot \sin A$ $= -\sin A$ $= -\frac{3}{7}$ </p>	<p>✓ $-\sin A$</p> <p>✓ $-\frac{3}{7}$</p> <p style="text-align: right;">(2)</p> <p>✓ $-\sin A$</p> <p>✓ $-\frac{3}{7}$</p> <p style="text-align: right;">(2)</p>
9.2		$\frac{\cos 100^\circ \times \tan^2 120^\circ}{\sin(-10^\circ)}$ $= \frac{(-\cos 80^\circ)(-\tan 60^\circ)^2}{(-\sin 10^\circ)}$ $= \frac{(-\cos 80^\circ) \times ((-\sqrt{3})^2)}{(-\cos 80^\circ)}$ $= 3$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Answer only: 0 marks</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If $\frac{+\cos 80^\circ}{+\sin 10^\circ}$ (assume two negatives cancelled), no penalty</p> </div>	<p>✓ $-\cos 80^\circ$</p> <p>✓ $-\tan 60^\circ$ or $\tan^2 60^\circ$</p> <p>✓ $-\sin 10^\circ$</p> <p>✓ $-\sqrt{3}$</p> <p>✓ $\sin 10^\circ = \cos 80^\circ$</p> <p>✓ 3</p> <p style="text-align: right;">(6)</p>

		<p style="text-align: center;">OR</p> $\frac{\cos 100^\circ \times \tan^2 120^\circ}{\sin(-10^\circ)}$ $= \frac{(-\cos 80^\circ)(-\tan 60^\circ)^2}{(-\sin 10^\circ)}$ $= \frac{(-\sin 10^\circ) \times ((-\sqrt{3})^2)}{(-\sin 10^\circ)}$ $= 3$ <p style="text-align: center;">OR</p> $\frac{\cos 100^\circ}{\sin(-10^\circ)} \times \tan^2 120^\circ$ $= \frac{\cos(90^\circ + 10^\circ)}{-\sin(10^\circ)} \times \tan^2 60^\circ$ $= \frac{-\sin 10^\circ}{-\sin 10^\circ} \times (\sqrt{3})^2$ $= 3$	<ul style="list-style-type: none"> ✓ $-\cos 80^\circ$ ✓ $-\sin 10^\circ$ ✓ $-\tan 60^\circ$ ✓ $-\sqrt{3}$ ✓ $\cos 80^\circ = \sin 10^\circ$ ✓ 3 <p style="text-align: right;">(6)</p>
		<ul style="list-style-type: none"> ✓ $\cos(90^\circ + 10^\circ)$ ✓ $-\sin 10^\circ$ ✓ $-\sin 10^\circ$ ✓ $\tan^2 60^\circ$ ✓ $\sqrt{3}$ ✓ 3 <p style="text-align: right;">(6)</p>	



9.3	9.3.1	$r = 5$ $\sin \hat{R}OP = \frac{3}{5} = 0,6$	<ul style="list-style-type: none"> ✓ 5 ✓ ratio <p style="text-align: right;">(2)</p>
	9.3.2	$\hat{R}OP = 36,87^\circ$ $\hat{Q}OP = 180^\circ - 36,869\dots^\circ$ $\hat{Q}OP = 143,13^\circ$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> Answer only: Full Marks </div> <ul style="list-style-type: none"> ✓ $36,869\dots^\circ$ ✓ $143,13^\circ$ <p style="text-align: right;">(2)</p>

<p>9.3.3</p>	$x_m = x \cos \theta + y \sin \theta$ $a = 4 \cos 115^\circ + 3 \sin 115^\circ$ $a = 1,03$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: Penalise 1 mark for rounding incorrectly Note: If incorrect angle is used in the x- formula: 1 mark</p> </div> <p style="text-align: center;">OR</p> <p>Rotation of 115° clockwise = 245° anticlockwise</p> $x_m = x \cos \theta - y \sin \theta$ $a = 4 \cos 245^\circ - 3 \sin 245^\circ$ $a = 1,03$ <p style="text-align: center;">OR</p> $\tan \hat{P\hat{O}R} = \frac{3}{4}$ $\hat{P\hat{O}R} = 36,86\dots^\circ$ $\hat{M\hat{O}R} = 78,13\dots^\circ$ $\cos \hat{M\hat{O}R} = \frac{a}{5}$ $a = 5 \cos 78,13^\circ$ $a = 1,03$	<p>✓ formula ✓ substitution of values ✓ $a = 1,03$ (3)</p> <p>✓ formula ✓ substitution of values ✓ $a = 1,03$ (3)</p> <p>✓ $36,86^\circ$</p> <p>✓ cos ratio</p> <p>✓ $a = 1,03$ (3)</p> <p style="text-align: right;">[18]</p>
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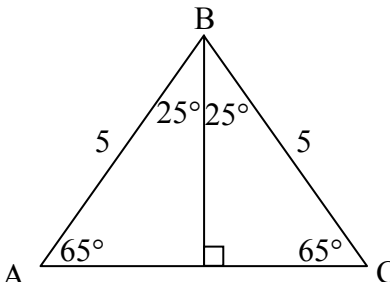
QUESTION 10

<p>10.1</p>	$f(225^\circ) = 2$ $\therefore a \tan 225^\circ = 2 \quad \therefore a = 2$ $g(0) = 4$ $\therefore b \cos 0^\circ = 4 \quad \therefore b = 4$	<div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Answer only: Full marks</p> </div>	<p>✓ substitution ✓ $a = 2$</p> <p>✓ substitution ✓ $b = 4$ (4)</p>
<p>10.2</p>	<p>Minimum value of $g(x) + 2 = -4 + 2 = -2$</p>	<div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Answer only: Full marks</p> </div>	<p>✓ -4 ✓ -2 (2)</p>
<p>10.3</p>	$\text{Period} = \frac{180^\circ}{\frac{1}{2}} = 360^\circ$	<div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Answer only: Full marks</p> </div>	<p>✓ $\frac{180^\circ}{\frac{1}{2}}$ ✓ 360° (2)</p>

10.4	<p>At P $f(\theta) = g(\theta)$ $2 \tan \theta = 4 \cos \theta$ for $180^\circ - \theta$: $2 \tan (180^\circ - \theta) = -2 \tan \theta$ and $4 \cos(180^\circ - \theta) = -4 \cos \theta$ $2 \tan \theta = 4 \cos \theta$ at P $\therefore -2 \tan \theta = -4 \cos \theta$ $\therefore 2 \tan (180^\circ - \theta) = 4 \cos (180^\circ - \theta)$ at Q</p> <p style="text-align: center;">OR</p> <p>$2 \tan \theta = 4 \cos \theta$ $\frac{\sin \theta}{\cos \theta} = 2 \cos \theta$ $\sin \theta = 2 \cos^2 \theta$ $= 2(1 - \sin^2 \theta)$ $2 \sin^2 \theta + \sin \theta - 2 = 0$ $\sin \theta = \frac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$ $\sin \theta = 0,78077\dots$ $\theta = 51,33^\circ$ or $128,67^\circ$ \therefore the x - coordinate of Q is $180^\circ - x_p$</p>	<p>$\checkmark 2 \tan \theta = 4 \cos \theta$ $\checkmark 2 \tan (180^\circ - \theta) = -2 \tan \theta$ $\checkmark 4 \cos(180^\circ - \theta) = -4 \cos \theta$ $\checkmark 2 \tan (180^\circ - \theta) = 4 \cos (180^\circ - \theta)$ (4)</p> <p>\checkmark equation</p> <p>$\checkmark \sin \theta = 0,78077\dots$ $\checkmark 51,33^\circ$ $\checkmark 128,67^\circ$ (4)</p> <p style="text-align: right;">[12]</p>
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QUESTION 11

11.1	<p>Area $\Delta ABC = \frac{1}{2} \cdot AB \cdot BC \cdot \sin 50^\circ$ $= \frac{1}{2} (5)(5) \sin 50^\circ$ $= 9,58 \text{ units}^2$</p> <p style="text-align: center;">OR</p> <p>Area of ΔABC $= \frac{1}{2} (2)(5) \sin 25^\circ (5 \cos 25^\circ)$ $= 9,58 \text{ units}^2$</p> <div style="text-align: center;"> <p>The diagram shows an isosceles triangle ABC with AB = BC = 5. The base angles are 65 degrees and the vertex angle is 50 degrees. A vertical line segment from B to the base AC is perpendicular to AC, forming two right-angled triangles with angles 25 degrees and 65 degrees.</p> </div> <p style="text-align: center;">OR</p> <p>Area of ΔABC $= [\frac{1}{2} (5 \cos 65^\circ)(5 \sin 65^\circ)](2)$ $= 9,58 \text{ units}^2$</p>	<p>\checkmark substitution into correct formula</p> <p>\checkmark answer (2)</p> <p>\checkmark base and height in terms of 5 and 25° \checkmark answer (2)</p> <p>\checkmark base and height in terms of 5 and 65° \checkmark answer (2)</p>
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<p>11.2</p>	<p> $AC^2 = 5^2 + 5^2 - 2(5)(5) \cos 50^\circ$ $AC^2 = 17,86061952$ $AC = 4,23 \text{ units}$ </p> <p style="text-align: center;">OR</p> <p> $\hat{A} = \hat{C} = 65^\circ$ (angles opposite equal sides) $\frac{\sin 65^\circ}{5} = \frac{\sin 50^\circ}{AC}$ $AC = \frac{5 \sin 50^\circ}{\sin 65^\circ}$ $= 4,23 \text{ units}$ </p> <p style="text-align: center;">OR</p> <p> $\sin 25^\circ = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5) \sin 25^\circ$ $= 4,23 \text{ units}$ </p> <div style="text-align: center;">  </div> <p style="text-align: center;">OR</p> <p> $\cos 65^\circ = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5) \cos 65^\circ$ $AC = 4,23 \text{ units}$ </p>	<p> ✓ use of cosine rule ✓ substitution ✓ answer (3) </p> <p> ✓ use of sine rule ✓ substitution ✓ answer (3) </p> <p> ✓ sketch/diagram ✓ $\sin 25^\circ = \frac{\frac{1}{2}AC}{5}$ ✓ answer (3) </p> <p> ✓ sketch/diagram ✓ $\cos 65^\circ = \frac{\frac{1}{2}(AC)}{5}$ ✓ answer (3) </p>
<p>11.3</p>	<p> $\tan 25^\circ = \frac{CF}{AC}$ $\therefore CF = 4,23 \times \tan 25^\circ$ $\therefore CF = 1,97 \text{ units}$ </p> <p style="text-align: center;">OR</p> <p> $\frac{FC}{\sin 25^\circ} = \frac{4,23}{\sin 65^\circ}$ $FC = \frac{4,23 \sin 25^\circ}{\sin 65^\circ}$ $= 1,97 \text{ units}$ </p>	<p> ✓ ratio ✓ CF as subject ✓ answer (3) </p> <p> ✓ sine rule ✓ FC as subject ✓ answer (3) </p> <p style="text-align: right;">[8]</p>

QUESTION 12

<p>12.1</p>	$LHS = \frac{\sin(360^\circ + 90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\sin(90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\cos(x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ <p style="text-align: center;">OR</p> $LHS = \frac{\sin[90^\circ - (\alpha - x)]}{\cos(\alpha - x)}$ $= \frac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ $= RHS$	<p>✓ subtracting 360° ✓ cos (x - α)</p> <p>✓ cos(α - x)</p> <p style="text-align: right;">(3)</p> <p>✓ subtracting 360° ✓ writing as 90° - (α - x) ✓ cos(α - x)</p> <p style="text-align: right;">(3)</p>
<p>12.2</p>	$\cos 2x = 1 - 3 \cos x$ $2 \cos^2 x - 1 = 1 - 3 \cos x$ $2 \cos^2 x + 3 \cos x - 2 = 0$ $(2 \cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -2$ <p style="text-align: center;">n/a</p> $x = 60^\circ + k.360^\circ ; k \in Z \quad \text{or} \quad x = 300^\circ + k.360^\circ ; k \in Z$ <p style="text-align: center;">OR</p> $x = \pm 60^\circ + k.360^\circ ; k \in Z$	<p>✓</p> $\cos 2x = 2 \cos^2 x - 1$ <p>✓ factorisation</p> $\cos x = \frac{1}{2}$ <p>✓ 60° ✓ 300° ✓ + k.360° ✓ k ∈ Z</p> <p style="text-align: right;">(7)</p>
<p>12.3.1</p>	<p>LHS:</p> $\frac{\sin A \cos B - \cos A \sin B}{\sin B \cos B}$ $= \frac{\sin(A - B)}{\sin B \cos B}$ <p>RHS =</p> $\frac{2 \sin(A - B)}{2 \sin B \cos B}$ $= \frac{\sin(A - B)}{\sin B \cos B}$ $= LHS$	<p>✓ writing as single fraction ✓ comp. angle expansion ✓ comp. angle expansion ✓ simplification</p> <p style="text-align: right;">(4)</p>

	<p>OR</p> <p>LHS:</p> $\frac{\sin A \cos B - \cos A \sin B}{\sin B \cos B}$ $= \frac{\sin(A - B)}{\sin B \cos B}$ $= \frac{2 \sin(A - B)}{2 \sin B \cos B}$ $= \frac{2 \sin(A - B)}{\sin 2B}$ $= RHS$	<p>✓ writing as single fraction ✓ comp. angle expansion ✓ mult. by 2 ✓ comp. angle expansion</p> <p style="text-align: right;">(4)</p>
	<p>OR</p> <p>RHS = $\frac{2 \sin(A - B)}{\sin 2B}$</p> $= \frac{2(\sin A \cos B - \cos A \sin B)}{2 \sin B \cos B}$ $= \frac{\sin A \cos B - \cos A \sin B}{\sin B \cos B}$ $= \frac{\sin A \cos B}{\sin B \cos B} - \frac{\cos A \sin B}{\sin B \cos B}$ $= \frac{\sin A}{\sin B} - \frac{\cos A}{\cos B}$ $= LHS$	<p>✓ expansion ✓ expansion ✓ divide by 2 ✓ write as separate fractions</p> <p style="text-align: right;">(4)</p>

	$\frac{1}{\sin 18^\circ} = 4 \cos 36^\circ$ $\frac{1}{\sin 18^\circ} = 4(1 - 2 \sin^2 18^\circ)$ $\frac{1}{\sin 18^\circ} = 4 - 8 \sin^2 18^\circ$ $8(\sin 18^\circ)^3 - 4(\sin 18^\circ) + 1 = 0$ <p>Hence $\sin 18^\circ$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$</p> <p>Note: substituting $x = \sin 18^\circ$ into $8x^3 - 4x + 1$ using a calculator showing equal to 0: 0 marks</p>	<p>✓ $\cos 36^\circ$ $= 1 - 2 \sin^2 18^\circ$ ✓ simplification ✓ equation i.t.o $\sin 18^\circ$ ✓ replacing $\sin 18^\circ = x$</p> <p style="text-align: right;">(4) [24]</p>
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TOTAL: 150



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2011

MARKS: 100

TIME: 2 hours

This question paper consists of 9 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round your answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. THREE diagram sheets for answering QUESTION 7.1, QUESTION 8.1, QUESTION 8.2, QUESTION 9, QUESTION 10 and QUESTION 11 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and present your work neatly.

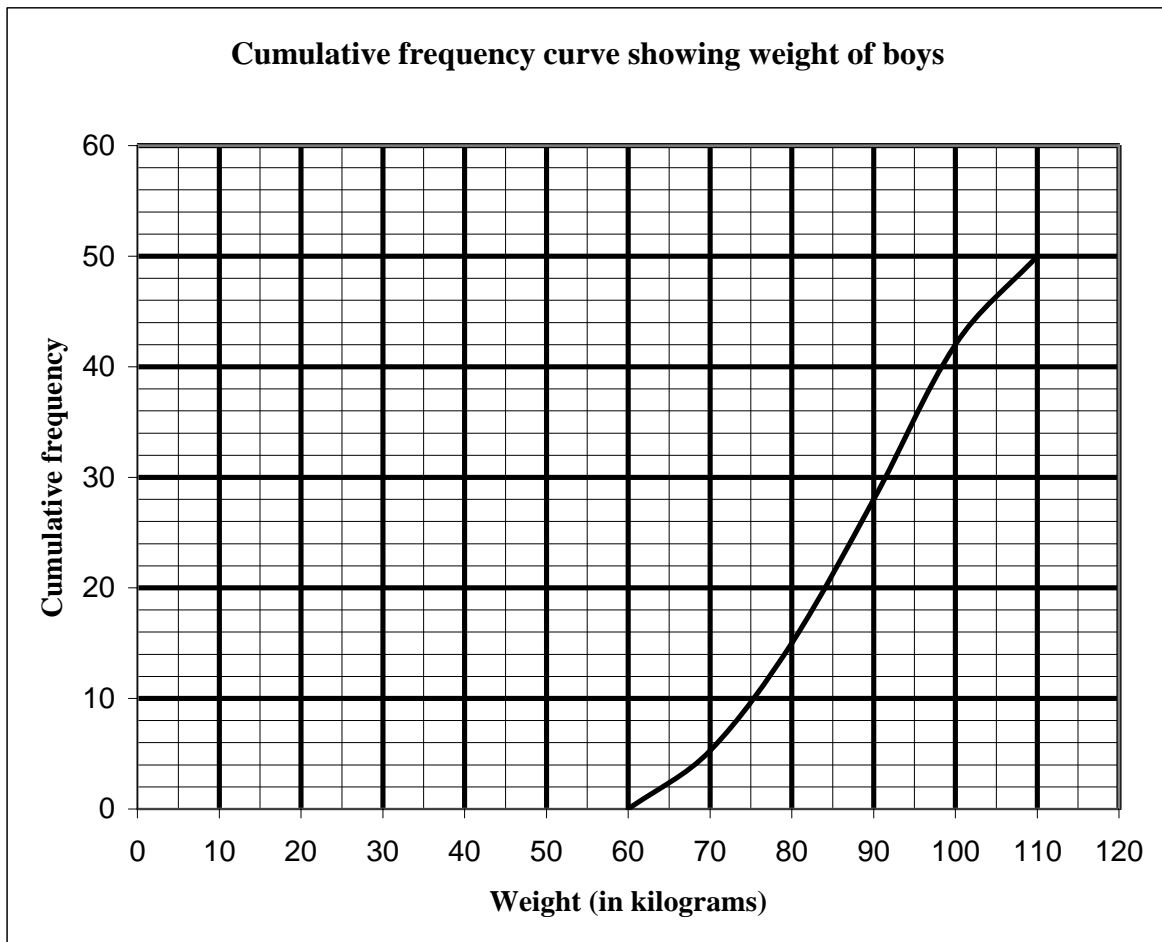
QUESTION 1

Consider the following recursive formula: $T_{k+1} = T_k - 2; k \geq 1; T_1 = 12$

- 1.1 Write down the first FOUR terms of the sequence. (3)
 - 1.2 How many terms of the above sequence must be added to give a sum of 0? (3)
- [6]**

QUESTION 2

The weights of a random sample of boys in Grade 11 were recorded. The cumulative frequency graph (ogive) represents the recorded weights.



- 2.1 How many of the boys weighed between 90 and 100 kilograms? (1)
 - 2.2 Estimate the median weight of the boys. (1)
 - 2.3 If there were 250 boys in Grade 11, estimate how many of them would weigh less than 80 kilograms? (2)
 - 2.4 It was suggested that the first 50 boys in Grade 11 to arrive at school on that day, be selected as a sample. Explain why this would not be a random sample. (1)
- [5]**

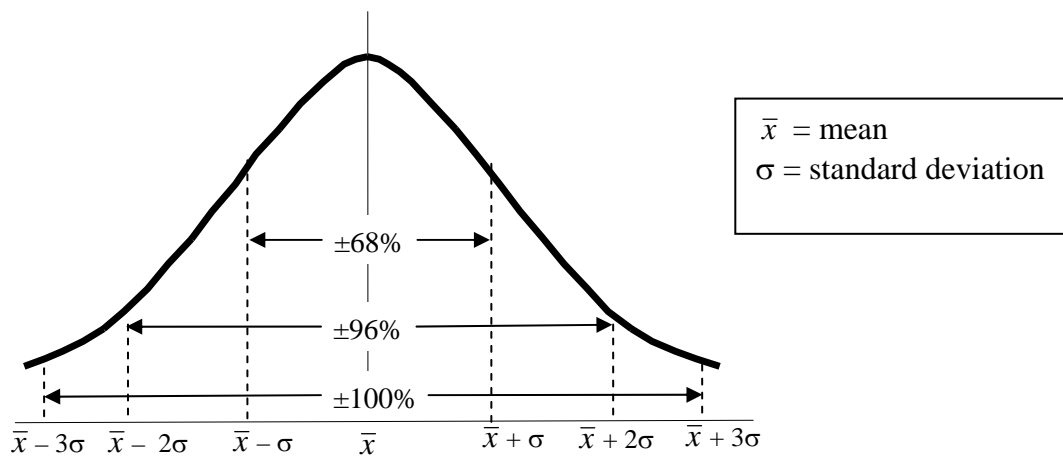
QUESTION 3

Let A and B be two events in a sample space. Suppose that $P(A) = 0,4$; $P(A \text{ or } B) = 0,7$ and $P(B) = k$.

- 3.1 For what value of k are A and B mutually exclusive? (2)
 - 3.2 For what value of k are A and B independent? (4)
- [6]**

QUESTION 4

The time taken for a pizza outlet to deliver to a customer is recorded. The data is found to be normally distributed with a mean time of 24 minutes and a standard deviation of 3 minutes.



Answer the following questions with reference to the information provided in the graph.

- 4.1 What percentage of pizzas are delivered between 21 and 24 minutes? (2)
 - 4.2 What percentage of pizzas are delivered between 15 and 27 minutes? (3)
 - 4.3 The outlet advertises that they will not charge for a pizza that takes longer than a certain time to deliver. If they want to give away no more than 2% of all deliveries, how many minutes should they allow for delivery? (3)
- [8]**

QUESTION 5

The digits 0, 1, 2, 3, 4, 5 and 6 are used to make 3 digit codes.

- 5.1 How many unique codes are possible if digits can be repeated? (2)
- 5.2 How many unique codes are possible if the digits cannot be repeated? (2)
- 5.3 In the case where digits may be repeated, how many codes are numbers that are greater than 300 and exactly divisible by 5? (3)
- [7]**

QUESTION 6

Complaints about a restaurant fell into three main categories: the menu (M), the food (F) and the service (S). In total 173 complaints were received in a certain month. The complaints were as follows:

- 110 complained about the menu.
- 55 complained about the food.
- 67 complained about the service.
- 20 complained about the menu and the food, but not the service.
- 11 complained about the menu and the service, but not the food.
- 16 complained about the food and the service, but not the menu.
- The number who complained about all three is unknown.

- 6.1 Draw a Venn diagram to illustrate the above information. (6)
- 6.2 Determine the number of people who complained about ALL THREE categories. (3)
- 6.3 Determine the probability that a complaint selected at random from those received, complained about AT LEAST TWO of the categories (that is. menu, food and service). (3)
- [12]**

QUESTION 7

The outdoor temperature, in °C, at noon on ten days and the number of units of electricity used to heat a house on each of those days, are shown in the table below.

Noon temperature (in °C)	7	11	9	2	4	7	0	10	5	3
Units of electricity used	32	20	27	37	32	28	41	23	33	36

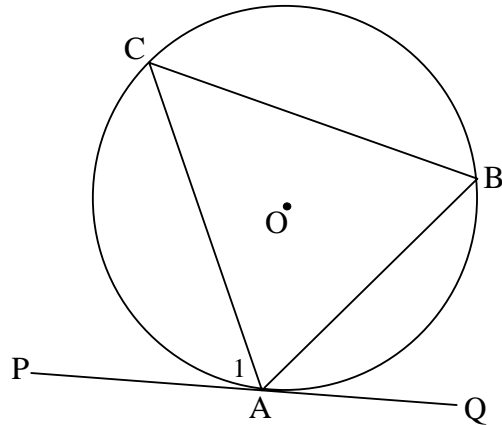
- 7.1 Draw a scatter graph that shows this information on the grid provided on DIAGRAM SHEET 1. (3)
- 7.2 Determine the equation of the least squares regression line. (4)
- 7.3 Determine the correlation coefficient. (2)
- 7.4 What can we conclude about the relationship between the noon temperature and the number of units of electricity used for heating? (2)
- 7.5 Estimate the number of units of electricity that was used to heat a house on a day when the outdoor temperature at noon was 8 °C. (2)
- [13]**

In the next FOUR questions, ensure you give reasons for each statement you make.

QUESTION 8

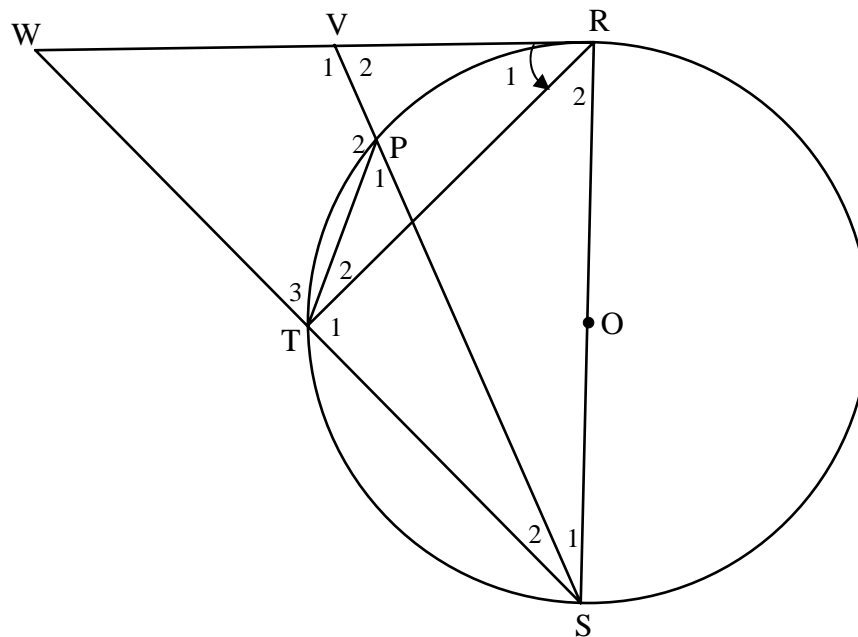
- 8.1 In the diagram below, O is the centre of the circle. PQ is a tangent to the circle at A. B and C are points on the circumference of the circle. AB, AC and BC are joined.

Prove the theorem that states $\hat{C}AP = \hat{A}BC$.



(5)

- 8.2 RS is a diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\hat{R}_1 = 50^\circ$.



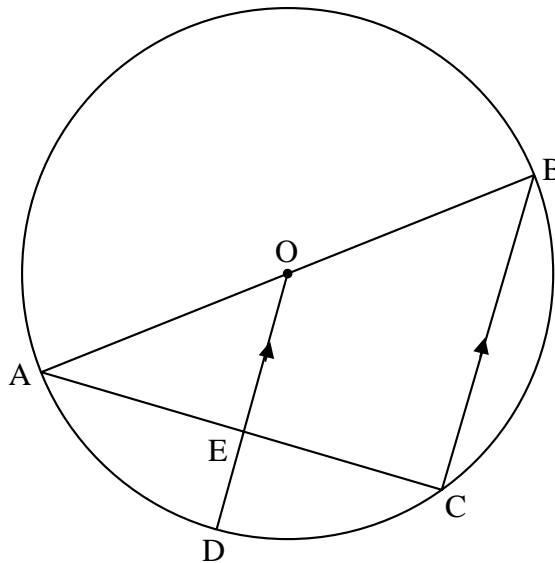
Calculate the size of:

- 8.2.1 $\hat{W}RS$ (1)
- 8.2.2 \hat{W} (2)
- 8.2.3 \hat{P}_1 (3)
- 8.2.4 Prove that $\hat{V}_1 = \hat{P}TS$. (4)

[15]

QUESTION 9

AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.

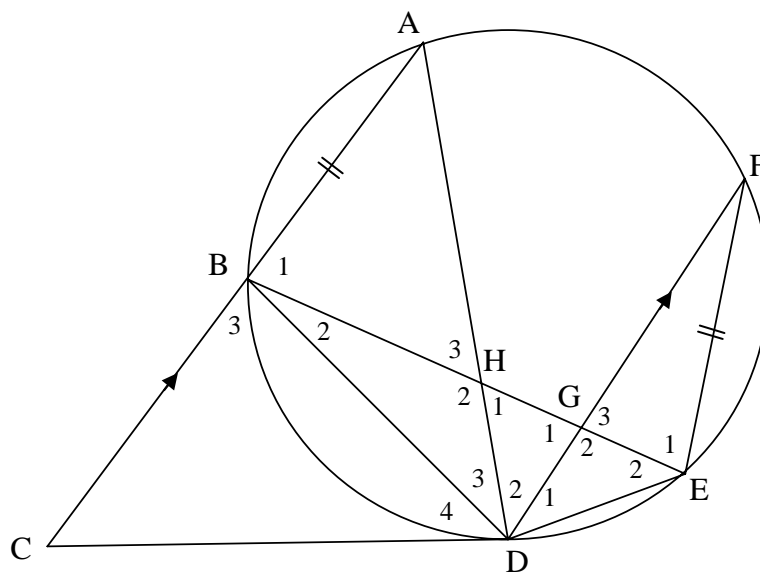


If the radius is 10 cm and $AC = 16$ cm, calculate the length of ED.

[5]

QUESTION 10

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. $AC \parallel FD$ and $FE = AB$. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.

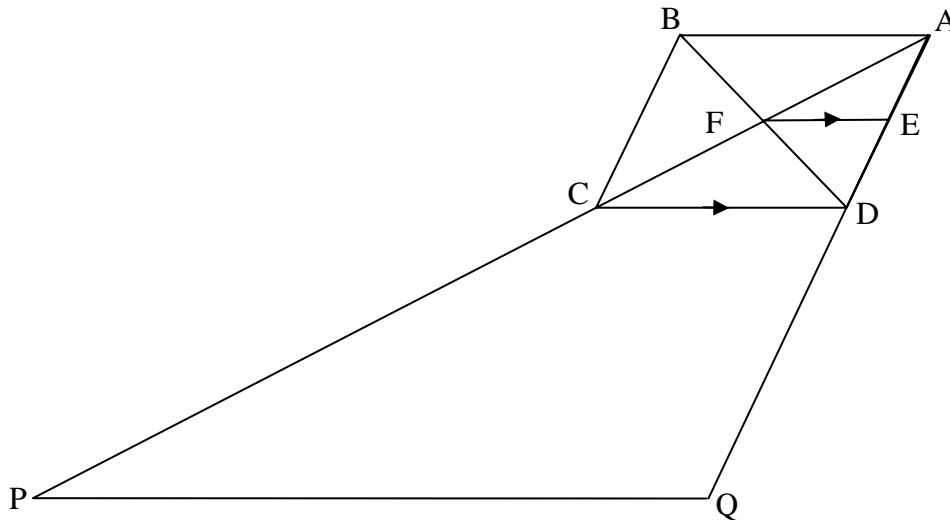


- 10.1 Determine THREE other angles that are each equal to x . (6)
- 10.2 Prove that $\triangle BHD \parallel \triangle FED$. (5)
- 10.3 Hence, or otherwise, prove that $AB \cdot BD = FD \cdot BH$. (2)

[13]

QUESTION 11

ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that $PC = 2AC$ and AD is produced to Q such that $DQ = 2AD$.



- 11.1 Show that E is the midpoint of AD. (2)
- 11.2 Prove $PQ \parallel FE$. (3)
- 11.3 If PQ is 60 cm, calculate the length of FE. (5)
- [10]**

TOTAL: 100

CENTRE NUMBER:

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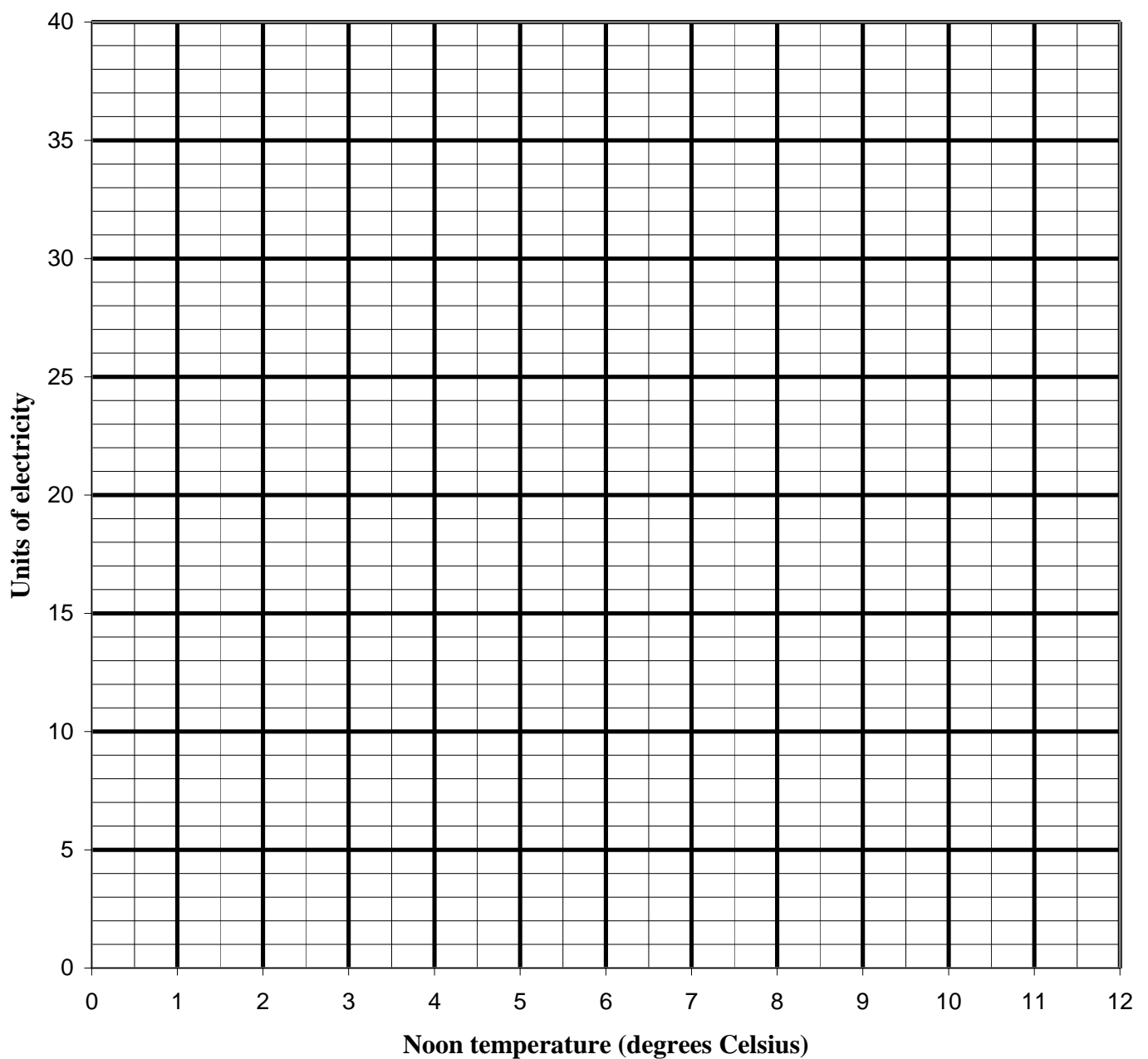
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DIAGRAM SHEET 1

QUESTION 7.1

Scatter plot showing noon temperature vs electricity consumption



CENTRE NUMBER:

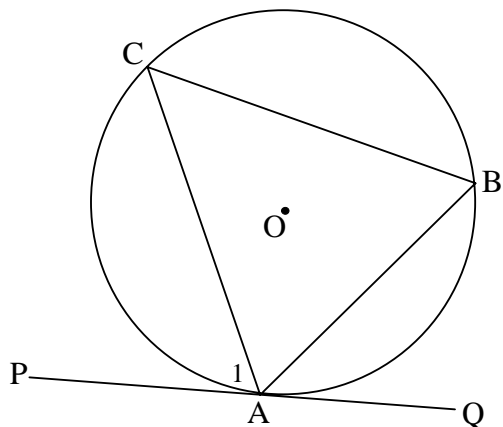
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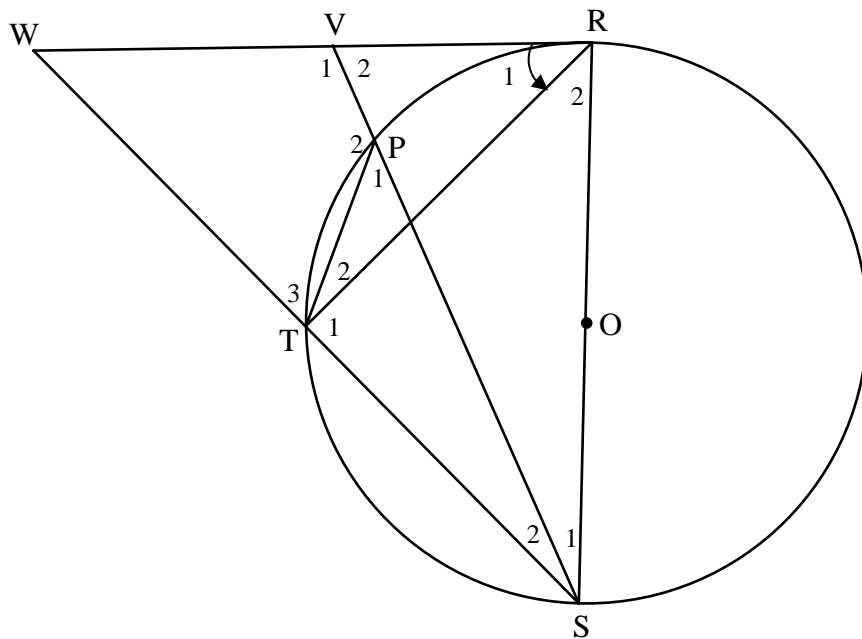
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DIAGRAM SHEET 2

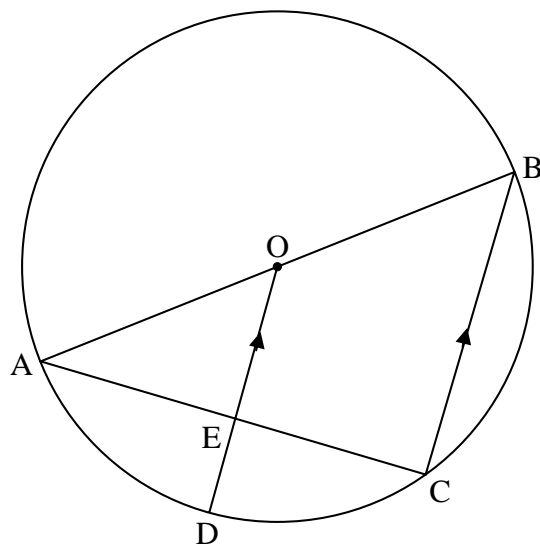
QUESTION 8.1



QUESTION 8.2



QUESTION 9



INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2011

MEMORANDUM

MARKS: 100

This memorandum consists of 14 pages.

NOTE:

- If a candidate answers a question TWICE and does not delete any attempt, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.
- A learner cannot use what s/he must prove to prove it (i.e. the circular argument.).

QUESTION 1

1.1	$T_{k+1} = T_k - 2; k \geq 1; T_1 = 12$ $T_1 = 12$ $T_2 = 12 - 2 = 10$ $T_3 = 10 - 2 = 8$ $T_4 = 8 - 2 = 6$	✓ 10 ✓ 8 ✓ 6 (3)
1.2	$12 + 10 + 8 + 6 + 4 + 2 + 0 + (-2) + (-4) + (-6) + (-8) + (-10) + (-12)$ $= 0$ <p>∴ 13 terms</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note: If a learner writes out $12 + 10 + 8 + 6 + 4 + 2 + 0$ then 1/3 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note: Answer only: FULL marks</p> </div> <p>OR</p> <p>There are 6 positive terms before the 7th term, which is 0. We need 6 negative terms of equal value to the positive terms so that the sum is zero</p> <p>6 positive terms + 1 zero term + 6 negative terms = 13 terms</p> <p>OR</p> $\frac{n}{2}[2(12) + (n-1)(-2)] = 0$ $\frac{n}{2}[24 + 2 - 2n] = 0$ $\frac{n}{2}[26 - 2n] = 0$ $13n - n^2 = 0$ $n(13 - n) = 0$ $n \neq 0 \quad \text{or} \quad n = 13$	✓✓ expansion ✓ 13 terms (3) ✓ $T_7 = 0$ ✓ 12 terms ✓ 13 terms (3) ✓ substitution into the arithmetic sum formula ✓ $\frac{n}{2}[26 - 2n] = 0$ ✓ 13 terms (3) [6]

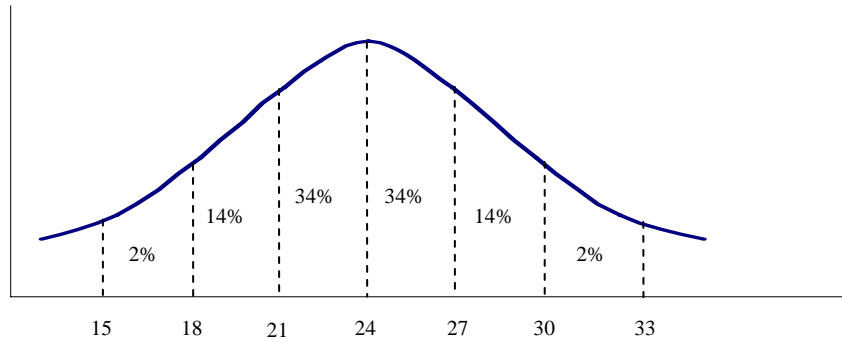
QUESTION 2

2.1	$42 - 28 = 14$	✓ answer (1)
2.2	Approximately 88 kg NOTE: Accept a range from 86 to 89 kg	✓ answer (1)
2.3	15 learners in the sample have a weight of less than 80 kg. One would expect $\frac{15}{50} \times 250 = 75$ learners in the grade to have a weight of less than 80 kg. OR 15 learners in the sample have a weight of less than 80 kg. One would expect $15 \times 5 = 75$ learners in the grade to have a weight of less than 80 kg. NOTE: <ul style="list-style-type: none"> Accept $\frac{14}{50} \times 250 = 70$ Answer as percentage: 1/2 marks Answer only: 2/2 marks 	✓ Cumulative Frequency value read off the graph when less than 80 ✓ answer (2) ✓ Cumulative Frequency value read off the graph when less than 80 ✓ answer (2)
2.4	This sampling method is biased towards those who arrive early on a Monday morning. In this way all the learners in the Grade do not have the same chance of being selected for the sample.	✓ sensible explanation of random sample (1) [5]

QUESTION 3

3.1	For mutually exclusive events $P(A \text{ or } B) = P(A) + P(B)$ $0,7 = 0,4 + k$ $k = 0,3$ NOTE: If the candidate writes down $k = 1 - 0,7 = 0,3$: 0/2 marks	Note: Answer only: FULL marks ✓ $0,7 = 0,4 + k$ ✓ answer (2)
3.2	For independent events $P(A \text{ and } B) = P(A) \cdot P(B)$ $= 0,4k$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,7 = 0,4 + k - 0,4k$ $0,3 = 0,6k$ $k = 0,5$ OR $0,7 = 0,4 + k - 0,4k$ $0,3 = 0,6k$ $k = 0,5$	Note: <ul style="list-style-type: none"> Answer only: 1/4 marks Wrong formula: 0/4 marks ✓ $P(A \text{ and } B) = P(A) \cdot P(B)$ ✓ $0,4k$ ✓ $0,7 = 0,4 + k - 0,4k$ ✓ answer (4) ✓✓✓ $0,7 = 0,4 + k - 0,4k$ ✓ answer (4) [6]

QUESTION 4



4.1	21 minutes is 1 standard deviation from the mean \therefore 34% of the pizzas are delivered between 21 and 24 minutes <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: Answer only: FULL marks </div>	✓ 1 standard deviation ✓ 34% (2)
4.2	15 minutes is 3 standard deviations to the left of the mean \therefore 50% 27 minutes is 1 standard deviation to the right of the mean \therefore 34% 84% of the pizzas are delivered between 15 and 27 minutes OR $2\% + 14\% + 34\% + 34\%$ $= 84\%$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: Answer only: FULL marks </div>	✓ 50% ✓ 34% ✓ 84% ✓ 50% ✓ 34% ✓ 84% (3)
4.3	The required 2% is the area found to the right of 2 standard deviations on the right hand side of the mean. Maximum for delivery should be $24 + 2(3)$ $= 30$ minutes <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: Answer only: FULL marks </div>	✓ 2 standard deviations ✓ $24 + 2(3)$ ✓ 30 (3) [8]

QUESTION 5

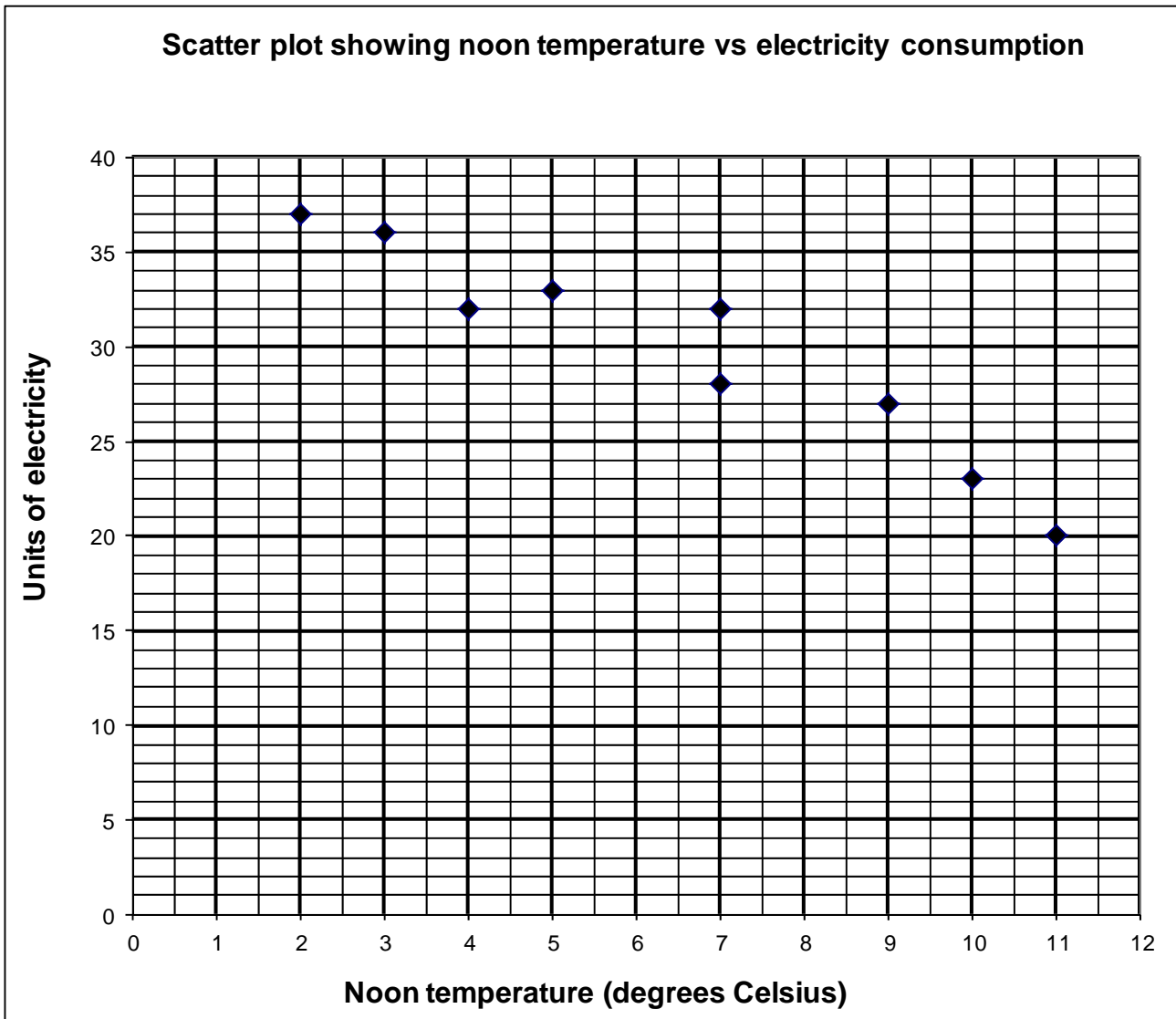
5.1	Number of unique codes $= 7 \times 7 \times 7$ $= 7^3$ $= 343$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: Answer only: FULL marks </div>	✓ $7 \times 7 \times 7$ ✓ answer (2)
5.2	Number of unique codes without repetition $= 7 \times 6 \times 5$ $= 210$ OR $\frac{7!}{4!}$ $= 210$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: Answer only: FULL marks </div>	✓ $7 \times 6 \times 5$ ✓ answer ✓ $\frac{7!}{4!}$ ✓ answer (2)
5.3	Number of codes with repetition that are greater than 300 and divisible by 5 $= 4 \times 7 \times 2 - 1$ $= 55$ OR For a 100 numbers there are 14 numbers divisible by 5 $14 \times 4 = 56$ $56 - 1 = 55$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Note: <ul style="list-style-type: none"> • No CA marking for the answer. • Answer only 3/3 marks </div>	✓ $4 \times 7 \times 2$ ✓ $- 1$ ✓ answer ✓ 14×4 ✓ $- 1$ ✓ answer (3) [7]

QUESTION 6

<p>6.1</p>		<p> ✓ $79 - x$ ✓ 20 ✓ $19 - x$ ✓ 11 ✓ 16 ✓ $40 - x$ </p> <p style="text-align: right;">(6)</p>
<p>6.2</p>	<p> $79 - x + 20 + x + 11 + 19 - x + 16 + 40 - x = 173$ $185 - 2x = 173$ $x = 6$ </p> <p>OR</p> <p>232 complaints and 173 people in total 94 complaints from 47 people 138 complaints from remaining 126 people For the two to be equal $126 - x = 138 - 3x$ $2x = 12$ $x = 6$</p> <p>OR</p> <p>$110 + 55 + 67 = 232$ $2x + 20 + 11 + 16 = 232 - 173$ $2x + 47 = 59$ $2x = 12$ $x = 6$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Check the reasonableness of the answer.</p> </div>	<p> ✓ addition ✓ 173 ✓ answer </p> <p style="text-align: right;">(3)</p> <p> ✓ $126 - x$ and $138 - 3x$ ✓ $126 - x = 138 - 3x$ ✓ answer </p> <p style="text-align: right;">(3)</p> <p> ✓ 232 ✓ $2x + 20 + 11 + 16 = 232 - 173$ ✓ answer </p> <p style="text-align: right;">(3)</p>
<p>6.3</p>	<p> P(at least two complaints) $= \frac{11 + 20 + 6 + 16}{173}$ $= \frac{53}{173}$ $= 0,31 \quad (0,30635838\dots)$ OR 30,64% </p>	<p> ✓ $11 + 20 + 6 + 16$ ✓ 173 </p> <p> ✓ answer </p> <p style="text-align: right;">(3) [12]</p>

QUESTION 7

Noon temperature (in °C)	2	3	4	5	7	7	9	10	11
Units of electricity used	37	36	32	33	32	28	27	23	20



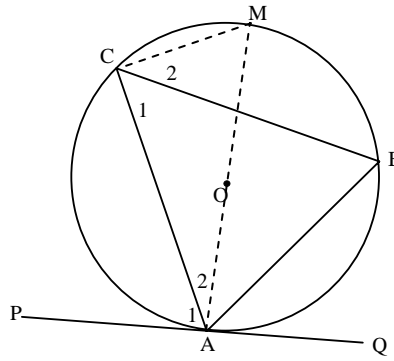
7.1	<p>See scatter plot above</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note: Please ignore the point (0 ; 41).</p> </div>	<p>✓✓✓ all 9 points plotted correctly 2 marks if 5 – 8 points are plotted correctly 1 mark if 1 – 4 points are plotted correctly.</p> <p style="text-align: right;">(3)</p>
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7.2	$a = 40,97$ (40,97108844...) $b = -1,74$ (-1,736394558...) $\hat{y} = 40,97 - 1,74x$ <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note:</p> <ul style="list-style-type: none"> • Penalise 1 mark for incorrect rounding to ONE decimal place in either 7.2 or 7.3 • Answer only: FULL marks </div> <p>NOTE: If the candidate works the coefficients out manually that $b = \frac{-204,2}{117,6}$ then 2 marks for b.</p>	✓✓ a ✓ b ✓ equation (4)
7.3	$r = -0,97$ (-0,9699269087...) <p>NOTE: If the candidate gives $b = \frac{6,139218}{3,42928}r$ and not simplified then 1 mark.</p>	✓✓ answer (2)
7.4	<p>There is a strong negative correlation between the noon temperature and the units of electricity used.</p> <p>OR</p> <p>As the noon temperature increases, the units of electricity used decreases.</p> <p>OR</p> <p>As the noon temperature decreases, the units of electricity used increases.</p>	✓ strong ✓ negative (2) ✓✓ as noon temp increases & units decrease (2) ✓✓ as noon temp decreases & units increases (2)
7.5	$\hat{y} \approx 40,97 - 1,74(8)$ $\approx 27,05$ <p>OR</p> $\hat{y} \approx 27,0799 \approx 27,08$ <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note:</p> <ul style="list-style-type: none"> • Answer only: 2/2 marks • Accept a range of 26,5 – 27,5 if the least squares regression line is drawn and the answer is read off: 2/2 marks </div>	✓ substitution ✓ answer (2) [13]

QUESTION 8

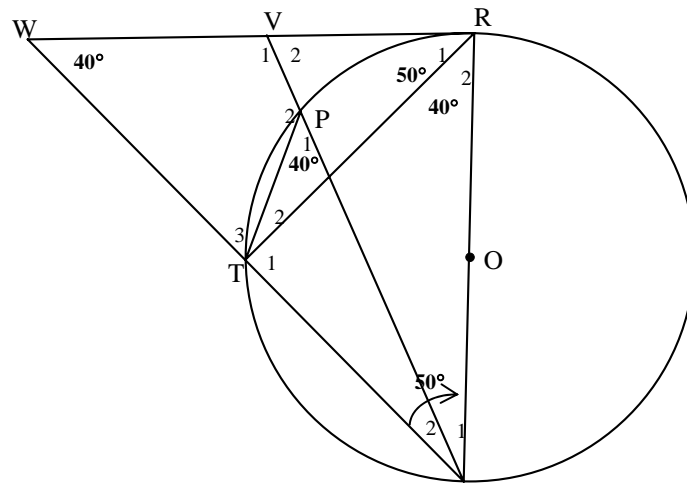
<p>8.1</p>	<p>Draw diameter AM and join M to B.</p> <p>$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (rad \perp tangent) $\hat{B}_1 + \hat{B}_2 = 90^\circ$ (\angles in a semi circle) $\hat{B}_2 = \hat{A}_2$ (\angles in same seg) $\hat{B}_1 = \hat{A}_1$</p> <p>OR</p> <p>Draw radii OC and OA Let $\hat{A}_2 = x$ $\hat{C}_1 = x$ (\angle opp = radii) $\hat{A}_1 = 90^\circ - x$ (rad \perp tan) $\hat{AOC} = 180^\circ - 2x$ (\angle sum Δ) $\hat{ABC} = 90^\circ - x$ (\angle circ cent = 2 \angle circumference) $\hat{ABC} = \hat{A}_1$ ($= 90^\circ - x$)</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>NOTE: If there is no construction: 0 / 5 marks</p> <p>If candidate changes lettering and states "Similarly": full marks</p> </div> <p>OR</p> <p>Draw QA extend to P. Draw tangent CP at C.</p> <p>PC = PA (tan from comm pt) $\hat{C}_2 = \hat{A}_1$ (\angles opp = sides) $\hat{COA} = 2\hat{ABC}$ (\angle circ cent = 2 \angle circumf) $\hat{A}_1 + \hat{A}_2 = 90^\circ$ (tan \perp radius) $\hat{COA} = 180^\circ - (90^\circ - \hat{A}_1 + 90^\circ - \hat{C}_2)$ $= \hat{A}_1 + \hat{C}_2$ $= \hat{A}_1 + \hat{A}_1$ $= 2\hat{A}_1$ $\hat{A}_1 = \frac{1}{2}\hat{COA}$ $= \hat{CBA}$</p>	<p>✓ construction ✓ S/R ✓ $\hat{B}_1 + \hat{B}_2 = 90^\circ$ ✓ \angles in a semi circle ✓ S/R</p> <p>(5)</p> <p>✓ construction ✓ $\hat{A}_1 = 90^\circ - x$ ✓ rad \perp tan ✓ S/R ✓ S/R</p> <p>(5)</p> <p>✓ construction ✓ S/R ✓ S/R ✓ $\hat{A}_1 + \hat{A}_2 = 90^\circ$ ✓ tan \perp radius</p> <p>(5)</p>
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Draw diameter AM and Join M and C
 $\widehat{MCA} = 90^\circ$ (\angle s in semi circle)
 $\widehat{AMC} + \widehat{A}_2 = 90^\circ$ (\angle sum Δ)
 $\widehat{A}_1 + \widehat{A}_2 = 90^\circ$ (rad \perp tangent)
 $\widehat{AMC} = \widehat{A}_1$
 $\widehat{AMC} = \widehat{B}$ (\angle s in same seg)
 $\widehat{A}_1 = \widehat{B}$



✓ construction
 ✓ S/R
 ✓ S/R
 ✓ $\widehat{A}_1 + \widehat{A}_2 = 90^\circ$
 ✓ tan \perp radius

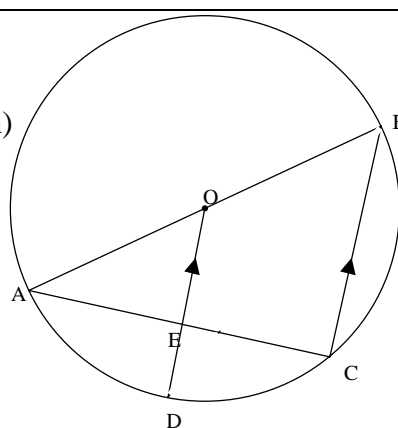
(5)



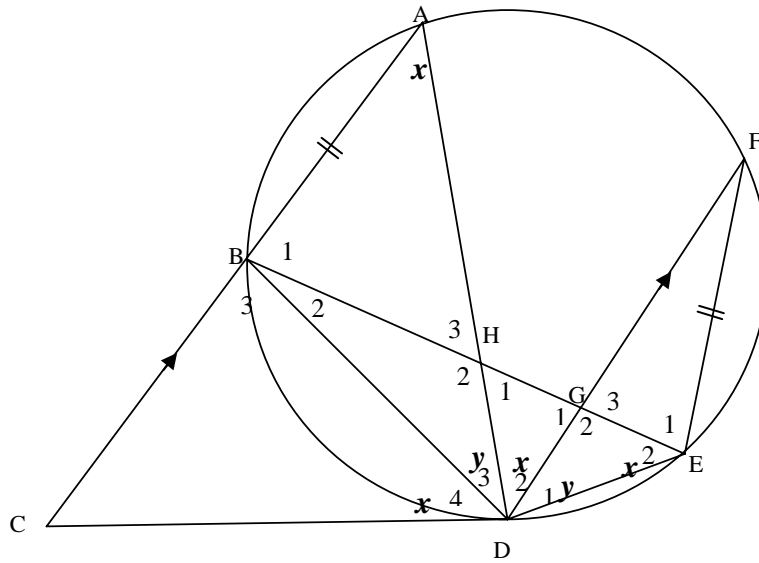
8.2.1	$\widehat{WRS} = 90^\circ$ (tan \perp radius)	✓ statement (1)
8.2.2	$\widehat{RST} = 50^\circ$ (tan ch th) $\widehat{W} = 40^\circ$ (\angle sum Δ) OR $\widehat{T}_1 = 90^\circ$ (\angle s in semi circle) $\widehat{W} + \widehat{R}_1 = \widehat{T}_1$ (ext \angle Δ) $\widehat{W} = 40^\circ$	✓ S/R ✓ $\widehat{W} = 40^\circ$ ✓ $\widehat{W} + \widehat{R}_1 = \widehat{T}_1$ ✓ $\widehat{W} = 40^\circ$ (2)
8.2.3	$\widehat{R}_2 = 40^\circ$ (tan \perp radius) $\widehat{P}_1 = 40^\circ$ (\angle s in same seg)	✓ $\widehat{R}_2 = 40^\circ$ ✓ $\widehat{P}_1 = 40^\circ$ ✓ \angle s in same seg (3)

8.2.4	<p>$\hat{P}_1 = \hat{W}$ ($= 40^\circ$) WVPT is a cyclic quadrilateral (ext $\angle =$ int opp) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ (ext \angle cyclic quad)</p> <p>OR $\hat{T}_1 = 90^\circ$ (\angles in semi circle) $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ $\hat{T}_2 = \hat{S}_1$ (\angles in same seg) $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{S}_1$ $\hat{V}_1 = 90^\circ + \hat{S}_1$ (ext \angle Δ) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$</p> <p>OR $\hat{P}_2 = 140^\circ$ (\angles on str line) $\hat{W} + \hat{P}_2 = 180^\circ$ WVPT is cyclic quad (opp \angles suppl) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ (ext \angle cyclic quad)</p> <p>OR $\hat{V}_1 = \hat{R}_1 + \hat{R}_2 + \hat{S}_1$ (ext \angle Δ) $\hat{V}_1 = 90^\circ + \hat{S}_1$ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ But $\hat{T}_2 = \hat{S}_1$ (\angles in same seg) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$</p> <p>OR In ΔPTS and ΔWVS $\hat{P}_1 = \hat{W}$ ($= 40^\circ$) \hat{S}_2 is common $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ (\angle sum Δ)</p>	<p>✓ $\hat{P}_1 = \hat{W}$ ✓ WVPT is a cyclic quadrilateral ✓ ext $\angle =$ in opp ✓ ext \angle cyclic quad (4)</p> <p>✓ \angles in semi circle ✓ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ \angles in same seg (4)</p> <p>✓ $\hat{W} + \hat{P}_2 = 180^\circ$ ✓ WVPT is a cyclic quadrilateral ✓ opp \angle suppl ✓ ext \angle cyclic quad (4)</p> <p>✓ $\hat{V}_1 = 90^\circ + \hat{S}_1$ ✓ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ \angles in same seg (4)</p> <p>✓ identification of triangles ✓ $\hat{P}_1 = \hat{W}$ ✓ \hat{S}_2 is common ✓ \angle sum Δ (4)</p> <p style="text-align: right;">[15]</p>
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QUESTION 9

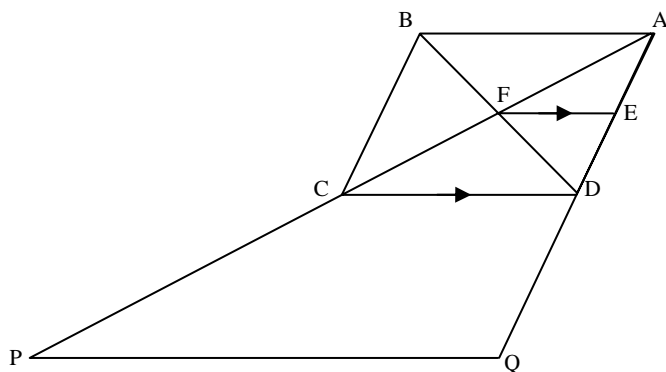
<p>9.</p>	<p> $\hat{C} = 90^\circ$ (\angles in semi circle) $\hat{OEA} = 90^\circ$ (corres \angles; $OD \parallel BC$) $AE = 8$ cm (line from circ cent \perp ch bis ch) $OE = 6$ cm (Pythagoras) $ED = 10 - 6 = 4$ cm OR $\hat{C} = 90^\circ$ (\angles in semi circle) $\hat{OEA} = 90^\circ$ (corres \angles; $OD \parallel BC$) $OE \parallel BC$ (given) $OA = OB$ (radii) $AE = EC = 8$ cm (midpoint theorem) $OE = 6$ cm (Pythagoras) $ED = 10 - 6 = 4$ cm OR $\hat{C} = 90^\circ$ (\angles in semi circle) $BC^2 = (20)^2 - (16)^2$ $BC^2 = 144$ $BC = 12$ $OE = \frac{1}{2} BC$ (midpoint theorem) $OE = 6$ cm $OD = 10$ cm $ED = 10 - 6 = 4$ cm OR $\hat{C} = 90^\circ$ (\angles in semi circle) $BC^2 = (20)^2 - (16)^2$ $BC^2 = 144$ $BC = 12$ $OE = \frac{1}{2} BC$ (midpoint theorem) $OE = 6$ cm $ED = 4$ cm </p>	 <p> $\checkmark \hat{C} = 90^\circ$ $\checkmark \hat{OEA} = 90^\circ$ \checkmark line from circ cent \perp ch bis ch $\checkmark OE = 6$ cm $\checkmark ED = 4$ cm $\checkmark \hat{C} = 90^\circ$ $\checkmark \hat{OEA} = 90^\circ$ \checkmark midpoint theorem $\checkmark OE = 6$ cm $\checkmark ED = 4$ cm $\checkmark \hat{C} = 90^\circ$ $\checkmark BC = 12$ \checkmark reason $\checkmark OE = 6$ cm $\checkmark ED = 4$ cm <p style="text-align: right;">[5]</p> </p>
		<p> $\checkmark \hat{C} = 90^\circ$ $\checkmark BC = 12$ \checkmark reason $\checkmark OE = 6$ cm $\checkmark ED = 4$ cm <p style="text-align: right;">[5]</p> </p>

QUESTION 10



<p>10.1</p>	<p>$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) OR (\angles in same seg) $\hat{D}_2 = \hat{A} = x$ (alt \angles; CA \parallel DF)</p>	<p>✓ $\hat{A} = x$ ✓ tan ch th ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ alt \angles; CA \parallel DF (6)</p>
<p>10.2</p>	<p>In $\triangle BHD$ and $\triangle FED$ 1. $\hat{B}_2 = \hat{F}$ (\angles in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = \angles) $\triangle BHD \parallel \triangle FED$ ($\angle\angle\angle$)</p>	<p>✓ $\hat{B}_2 = \hat{F}$ ✓ \angles in same seg ✓ $\hat{D}_3 = \hat{D}_1$ ✓ = chs subt = \angles ✓ $\angle\angle\angle$ (5)</p>
<p>10.3</p>	<p>$\frac{FE}{BH} = \frac{FD}{BD}$ ($\parallel \Delta$s) But $FE = AB$ (given) $\frac{AB}{BH} = \frac{FD}{BD}$ $AB \cdot BD = FD \cdot BH$</p>	<p>✓ $\frac{FE}{BH} = \frac{FD}{BD}$ ✓ $FE = AB$ (2) [13]</p>

QUESTION 11



11.1	$AF = FC$ $FE \parallel CD$ $AE = ED$ (diags of parallelogram bisect) (Prop Th; $FE \parallel CD$) OR (Midpoint Theorem)	✓ $AF = FC$ ✓ reason (2)
11.2	$\frac{AC}{CP} = \frac{1}{2}$ (given) $\frac{AD}{DQ} = \frac{1}{2}$ (given) $\frac{AC}{CP} = \frac{AD}{DQ}$ $CD \parallel PQ$ (converse proportionality theorem) $CD \parallel FE$ (given) $\therefore PQ \parallel FE$ OR $\frac{AC}{AP} = \frac{1}{3}$ $\frac{AD}{AQ} = \frac{1}{3}$ $\frac{AC}{AP} = \frac{AD}{AQ}$ $CD \parallel PQ$ (converse proportionality theorem) $CD \parallel FE$ (given) $\therefore PQ \parallel FE$ OR $\frac{AF}{AP} = \frac{1}{6}$ $\frac{AE}{AQ} = \frac{1}{6}$ $\frac{AF}{AP} = \frac{AE}{AQ}$ $\therefore PQ \parallel FE$ (converse proportionality theorem)	✓ ratios equal ✓ $CD \parallel PQ$ ✓ reason: converse prop th and conclusion (3) ✓ ratios equal ✓ $CD \parallel PQ$ ✓ reason: converse prop th and conclusion (3) ✓ $\frac{AF}{AP} = \frac{1}{6}$ ✓ $\frac{AF}{AP} = \frac{AE}{AQ}$ ✓ conv prop theorem

11.3	<p>In $\triangle AEF$ and $\triangle APQ$</p> <ol style="list-style-type: none"> \hat{A} is common $\hat{A}\hat{E}F = \hat{A}\hat{Q}P$ (corres \angles; $FE \parallel PQ$) $\hat{A}\hat{F}E = \hat{A}\hat{P}Q$ (corres \angles; $FE \parallel PQ$) <p>$\therefore \triangle AEF \parallel \triangle APQ$ ($\angle\angle\angle$)</p> $\frac{FE}{PQ} = \frac{AF}{AP} \quad (\parallel \Delta\text{s})$ $\frac{FE}{60} = \frac{1}{6}$ <p>$FE = 10$ cm</p> <p>OR</p> <p>In $\triangle ADC$ and $\triangle APQ$</p> <ol style="list-style-type: none"> \hat{A} is common $\hat{A}\hat{D}C = \hat{A}\hat{Q}P$ (corres \angles; $CD \parallel PQ$) $\hat{A}\hat{C}D = \hat{A}\hat{P}Q$ (corres \angles; $CD \parallel PQ$) <p>$\therefore \triangle ADC \parallel \triangle APQ$ ($\angle\angle\angle$)</p> $\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3} \quad (\parallel \Delta\text{s})$ $CD = \frac{1}{3} PQ$ <p>$CD = 20$ cm But $AF = FC$ $AE = ED$ (Midpoint Theorem)</p> $FE = \frac{1}{2} CD$ <p>$FE = 10$ cm</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>NOTE: If the similarity has not been proven, then max 3/5 marks</p> </div>	<p>✓ first pair of angles equal with reason ✓ second pair of angles equal with reason</p> <p>✓ $\frac{AF}{AP} = \frac{1}{6}$ ✓ $\frac{FE}{PQ} = \frac{AF}{AP}$ ✓ answer (5)</p> <p>✓ first pair of angles equal with reason ✓ second pair of angles equal with reason</p> <p>✓ $CD = \frac{1}{3} PQ$</p> <p>✓ $FE = \frac{1}{2} CD$ ✓ answer (5)</p> <p style="text-align: right;">[10]</p>
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TOTAL: 100