



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2010

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 2 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. TWO diagram sheets for answering QUESTION 5.3, QUESTION 6.4 and QUESTION 11.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x , correct to TWO decimal places, where necessary:

1.1.1 $(3 - x)(5 - x) = 3$ (3)

1.1.2 $3x^2 = 2(x + 2)$ (4)

1.1.3 $4 + 5x > 6x^2$ (4)

1.2 Solve for x and y simultaneously:

$$3y = 2x$$

$$x^2 - y^2 + 2x - y = 1$$
 (7)

1.3 Calculate the integer that is the closest approximation to:

$$\frac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}} \quad (\text{Show ALL workings.})$$
 (3)
[21]

QUESTION 2

2.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$ (4)

2.2 The following sequence forms a convergent geometric sequence: $5x ; x^2 ; \frac{x^3}{5} ; \dots$

2.2.1 Determine the possible values of x . (3)

2.2.2 If $x = 2$, calculate S_{∞} . (2)

2.3 The following arithmetic sequence is given: $20 ; 23 ; 26 ; 29 ; \dots ; 101$

2.3.1 How many terms are there in this sequence? (2)

2.3.2 The even numbers are removed from the sequence.
Calculate the sum of the terms of the remaining sequence. (6)
[17]

QUESTION 3

The sequence $4 ; 9 ; x ; 37; \dots$ is a quadratic sequence.

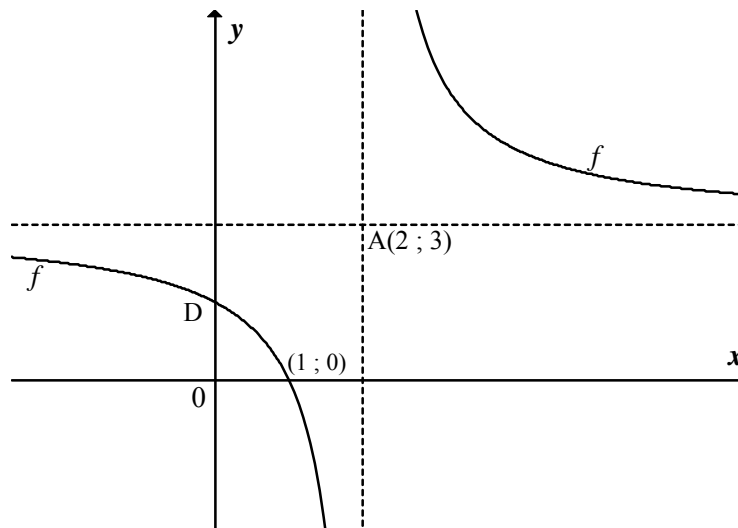
- 3.1 Calculate x . (3)
- 3.2 Hence, or otherwise, determine the n^{th} term of the sequence. (4)
[7]

QUESTION 4

Given $f(x) = \frac{a}{x-p} + q$. The point $A(2 ; 3)$ is the point of intersection of the asymptotes of f .

The graph of f intersects the x -axis at $(1 ; 0)$.

D is the y -intercept of f .



- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Determine an equation of f . (3)
- 4.3 Write down the coordinates of D . (2)
- 4.4 Write down an equation of g if g is the straight line joining A and D . (3)
- 4.5 Write down the coordinates of the other point of intersection of f and g . (4)
[14]

QUESTION 5

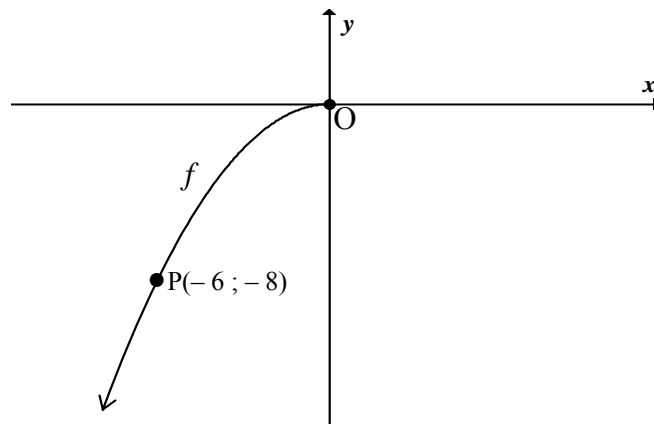
Consider the function $f(x) = 4^{-x} - 2$.

- 5.1 Calculate the coordinates of the intercepts of f with the axes. (4)
- 5.2 Write down the equation of the asymptote of f . (1)
- 5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)
- 5.4 Write down the equation of g if g is the graph of f shifted 2 units upwards. (1)
- 5.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.) (3)
- [12]**

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.

The point $P(-6; -8)$ lies on the graph of f .



- 6.1 Calculate the value of a . (2)
- 6.2 Determine the equation of f^{-1} , in the form $y = \dots$ (3)
- 6.3 Write down the range of f^{-1} . (1)
- 6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from $(0; 0)$. (2)
- 6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x -axis. Determine the equation of the new function in the form $y = \dots$ (3)
- [11]**

QUESTION 7

- 7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5)
- 7.2 Timothy buys furniture to the value of R10 000. He borrows the money on 1 February 2010 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Timothy agrees to pay monthly instalments of R450. The agreement of the loan allows Timothy to start paying these equal monthly instalments from 1 August 2010.
- 7.2.1 Calculate the total amount owing to the financial institution on 1 July 2010. (2)
- 7.2.2 How many months will it take Timothy to pay back the loan? (4)
- 7.2.3 What is the balance of the loan immediately after Timothy has made the 25th payment? (3)
- [14]**

QUESTION 8

- 8.1 Differentiate $g(x) = x^2 - 5$ from first principles. (5)
- 8.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{x^6}{2} + 4\sqrt{x}$. (3)
- 8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b . (6)
- [14]**

QUESTION 9

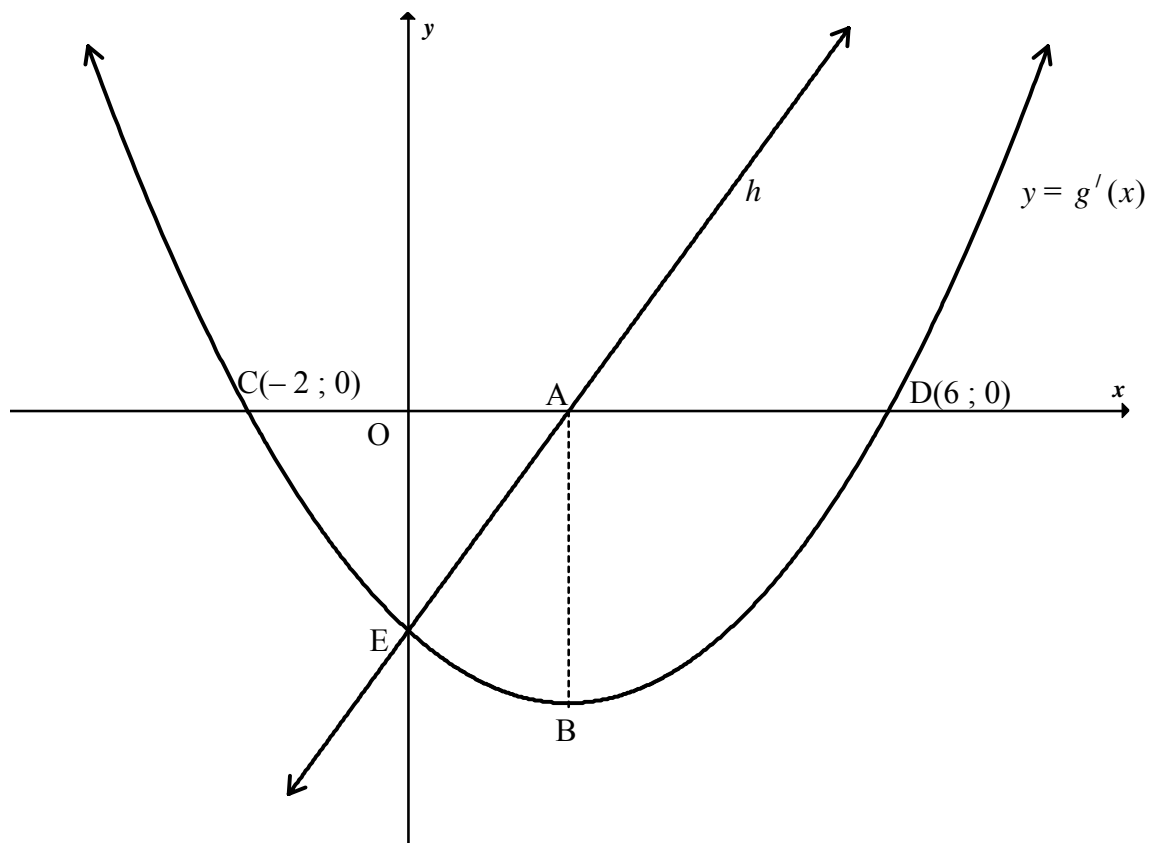
The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g .

The graphs of h and g' have a common y -intercept at E .

$C(-2; 0)$ and $D(6; 0)$ are the x -intercepts of the graph of g' .

A is the x -intercept of h and B is the turning point of g' .

$AB \parallel y$ -axis.

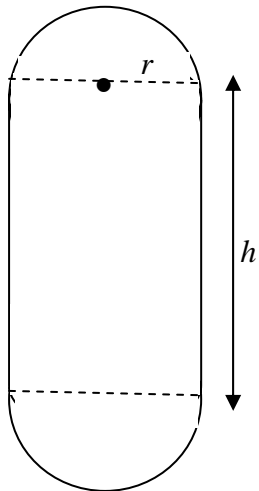


- 9.1 Write down the coordinates of E . (1)
- 9.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. (4)
- 9.3 Write down the x -coordinates of the turning points of g . (2)
- 9.4 Write down the x -coordinate of the point of inflection of the graph of g . (2)
- 9.5 Explain why g has a local maximum at $x = -2$. (3)
- [12]**

QUESTION 10

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$ Curved surface area of a cylinder = $2\pi rh$ Volume of a sphere = $\frac{4}{3}\pi r^3$ Volume of a cylinder = $\pi r^2 h$

10.1 Show that $h = \frac{1}{6r^2} - \frac{4r}{3}$. (3)

10.2 Hence, show that the outer surface area of the satellite can be given as $S = \frac{4\pi r^2}{3} + \frac{\pi}{3r}$. (3)

10.3 Calculate the minimum outer surface area of the satellite. (6)
[12]

QUESTION 11

A factory produces two types of braai stands, Type **A** and Type **B**.

- Type A requires one hour of machine-time and three hours for welding and finishing.
- Type B requires two hours of machine-time and one hour for welding and finishing.
- In one day the factory has available no more than 28 hours machine-time and no more than 24 hours for welding and finishing.

- 11.1 If the factory produces x Type A and y Type B braai stands on a particular day, write down the relevant constraints in terms of x and y . (4)
- 11.2 Represent the system of constraints on the graph paper provided on DIAGRAM SHEET 2. Indicate the feasible region by means of shading. (3)
- 11.3 Now determine the largest number of the following types that could be manufactured in one day:
- 11.3.1 Type A (1)
- 11.3.2 Type B (1)
- 11.4 Determine how many Type A and Type B braai stands should be manufactured each day for the factory to produce the maximum number of braai stands. (2)
- 11.5 If the demand for Type A braai stands is at least as large as the demand for Type B braai stands, calculate the largest number of braai stands that can be manufactured in one day and the machine-time required in this case. (5)

[16]**TOTAL: 150**

CENTRE NUMBER:

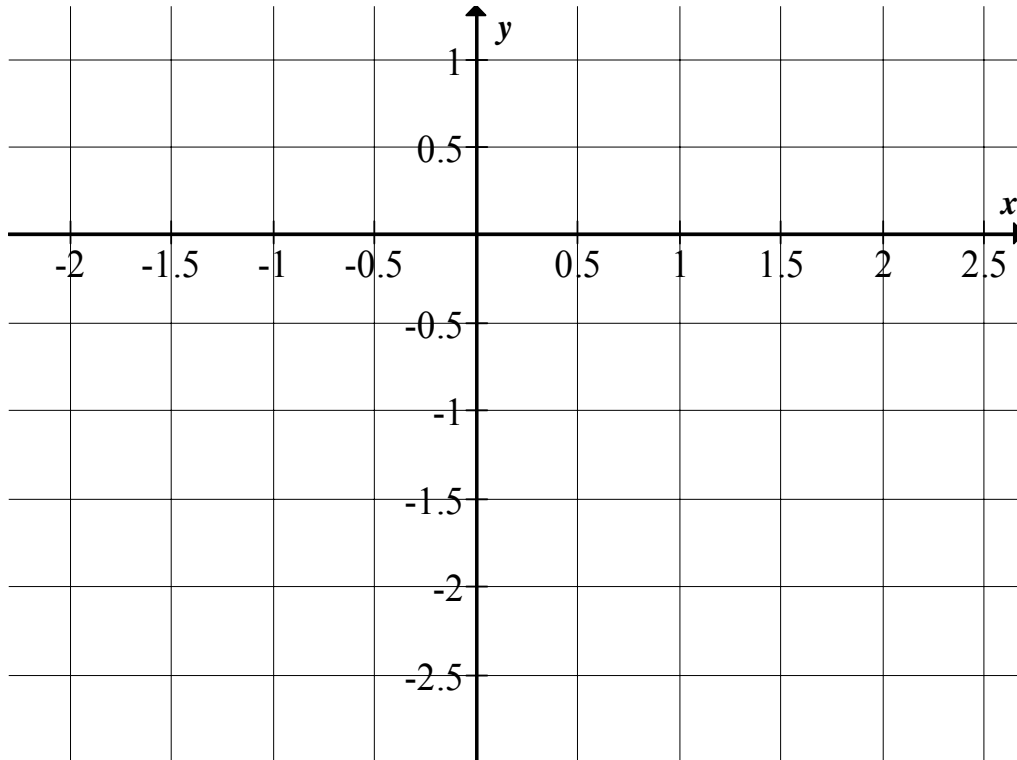
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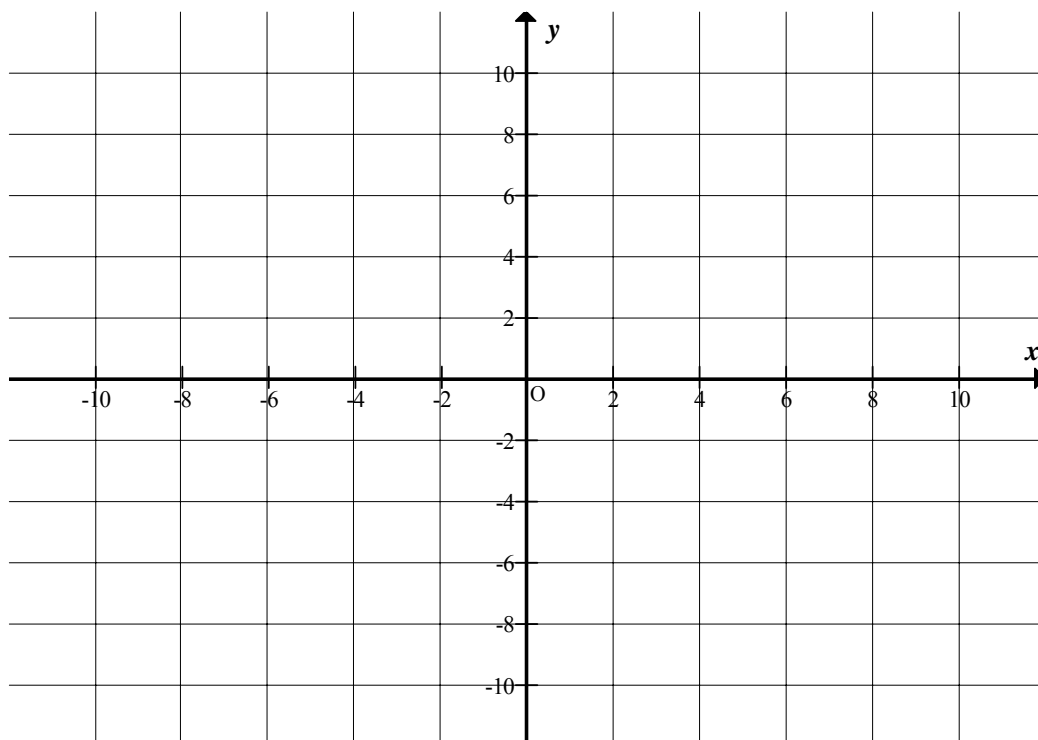
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DIAGRAM SHEET 1

QUESTION 5.3



QUESTION 6.4



CENTRE NUMBER:

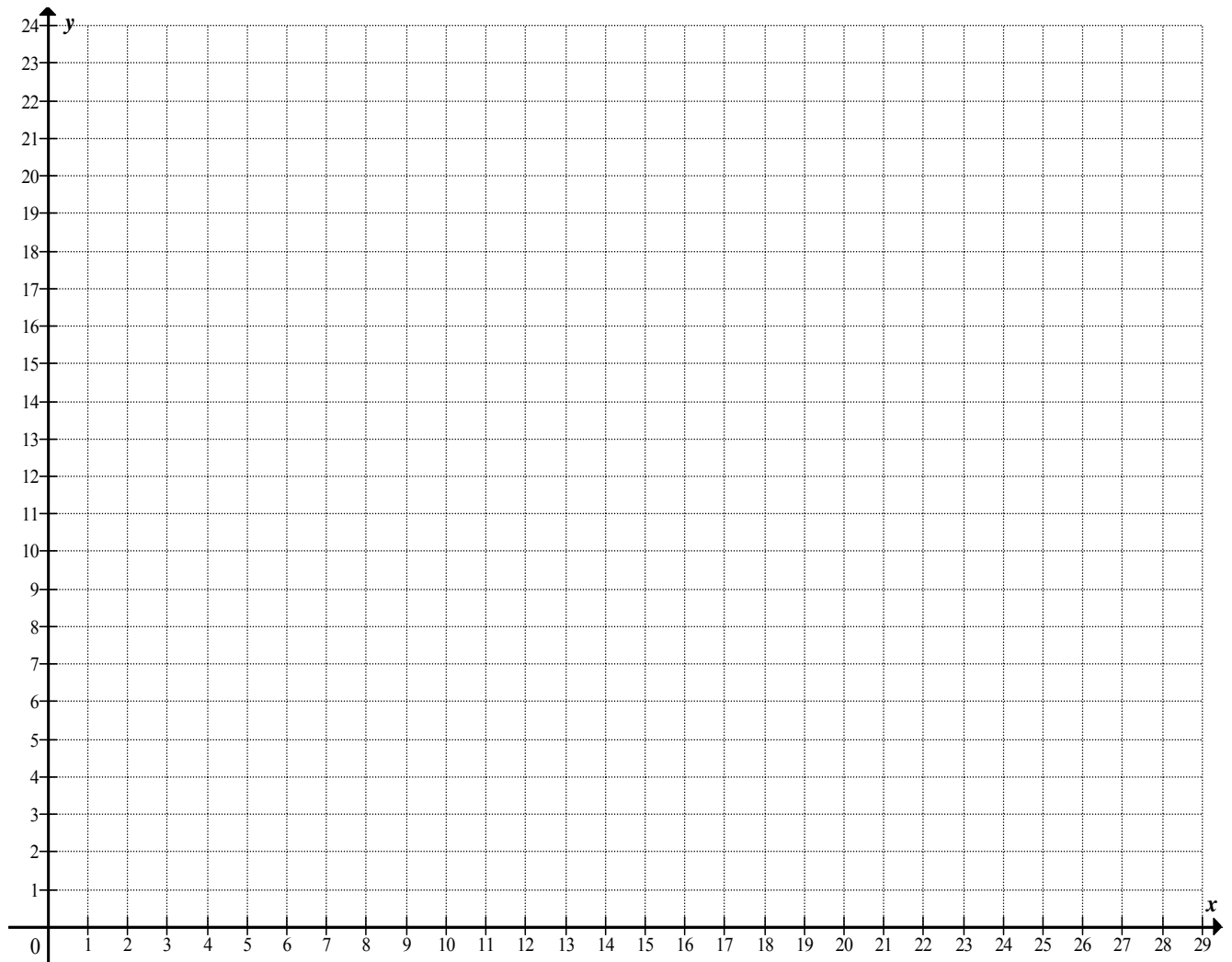
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DIAGRAM SHEET 2

QUESTION 11.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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GRADE 12

MATHEMATICS P1

NOVEMBER 2010

MEMORANDUM

MARKS: 150

This memorandum consists of 27 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

QUESTION 1

<p>1.1.1</p>	$(3 - x)(5 - x) = 3$ $15 - 8x + x^2 = 3$ $x^2 - 8x + 12 = 0$ $(x - 6)(x - 2) = 0$ $x = 6 \text{ or } x = 2$ <p>OR</p> $(3 - x)(5 - x) = 3$ $15 - 8x + x^2 = 3$ $x^2 - 8x + 12 = 0$ $(x - 4)^2 = 4$ $x - 4 = 2 \text{ or } x - 4 = -2$ $x = 6 \text{ or } x = 2$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: If answer only : Full Marks</p> <p>If the candidate makes it a linear equation, no marks</p> <p>For only 1 answer: 1 / 3</p> </div>	<p>✓ expansion ✓ factors ✓ answers (3)</p> <p>✓ expansion ✓ completed square form ✓ answers (3)</p>
<p>1.1.2</p>	$3x^2 = 2(x + 2)$ $3x^2 - 2x - 4 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$ $= \frac{2 \pm \sqrt{52}}{6}$ $x = 1,54 \text{ or } -0,87$ <p>OR</p> $3x^2 = 2(x + 2)$ $3x^2 - 2x - 4 = 0$ $x^2 - \frac{2}{3}x = \frac{4}{3}$ $\left(x - \frac{1}{3}\right)^2 = \frac{4}{3} + \frac{1}{9}$ $\left(x - \frac{1}{3}\right)^2 = \frac{13}{9}$ $x - \frac{1}{3} = \pm \frac{\sqrt{13}}{3}$ $x = \frac{1 \pm \sqrt{13}}{3}$ $x = 1,54 \text{ or } -0,87$	<div style="border: 1px solid black; padding: 5px;"> <p>Note: No penalty for incorrect rounding off of answers.</p> <p>Substitution into incorrect formula, no marks</p> </div>	<p>✓ standard form ✓✓ substitution ✓ answers (4)</p> <p>✓ expansion ✓ completed square ✓ $\pm \frac{\sqrt{13}}{3}$ ✓ answers (4)</p>

<p>1.1.3</p>	<p> $4 + 5x > 6x^2$ $0 > 6x^2 - 5x - 4$ $0 > (3x - 4)(2x + 1)$ critical values: $x = \frac{5 \pm \sqrt{121}}{12}$ $x = -\frac{1}{2}$ or $\frac{4}{3}$ </p> <p> $+ \quad 0 \quad - \quad 0 \quad +$ $\frac{-\frac{1}{2}}{\quad} \quad \frac{4}{3}$ </p> <p> $-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$ OR $-\frac{1}{2} < x$ and $x < \frac{4}{3}$ </p> <p>OR</p> <p> $-6x^2 + 5x + 4 > 0$ $(-3x + 4)(2x + 1) > 0$ critical values: $-\frac{1}{2}$ and $\frac{4}{3}$ </p> <p> $- \quad 0 \quad + \quad 0 \quad -$ $\frac{-\frac{1}{2}}{\quad} \quad \frac{4}{3}$ </p> <p> $-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$ OR $-\frac{1}{2} < x$ and $x < \frac{4}{3}$ </p>	<p> ✓ correct inequality ✓ factors ✓ critical values $-\frac{1}{2}$ and $\frac{4}{3}$ </p> <p> ✓ answer (4) </p> <p> ✓ correct inequality ✓ factors ✓ critical values $-\frac{1}{2}$ and $\frac{4}{3}$ </p> <p> ✓ answer (4) </p>
<p>1.2</p>	<p> $3y = 2x$ $y = \frac{2x}{3}$ $x^2 - \left(\frac{2x}{3}\right)^2 + 2x - \left(\frac{2x}{3}\right) = 1$ OR $x^2 - \left(\frac{2x}{3}\right)^2 + 2x - \left(\frac{2x}{3}\right) = 1$ $x^2 - \frac{4x^2}{9} + 2x - \frac{2x}{3} = 1$ $9x^2 - 4x^2 + 18x - 6x = 9$ $5x^2 + 12x - 9 = 0$ $(5x - 3)(x + 3) = 0$ $x = \frac{3}{5}$ or $x = -3$ </p> <p> $\frac{5x^2}{9} + \frac{4x}{3} - 1 = 0$ $x = \frac{-\frac{4}{3} \pm \sqrt{\left(\frac{4}{3}\right)^2 - 4\left(\frac{5}{9}\right)(-1)}}{2\left(\frac{5}{9}\right)}$ $= \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} + \frac{20}{9}}}{\frac{10}{9}}$ $x = 0,6$ or $x = -3$ </p>	<p> ✓ $y = \frac{2x}{3}$ ✓ substitution ✓ simplification ✓ standard forms ✓ factors or substitution into correct formula ✓ x-answers ✓ y-answers </p>

	<p> $y = \frac{2}{5}$ or $y = -2$ </p> <p> $(x ; y) = \left(\frac{3}{5}; \frac{2}{5}\right)$ or $(-3 ; -2)$ </p> <p>OR</p> <p> $3y = 2x$ </p> <p> $x^2 - y^2 + 2x - y = 1$ </p> <p> $4x^2 - 4y^2 + 8x - 4y = 4$ </p> <p> $(2x)^2 - 4y^2 + 8x - 4y = 4$ </p> <p> $(3y)^2 - 4y^2 + 4(3y) - 4y = 4$ </p> <p> $9y^2 - 4y^2 + 8y = 4$ </p> <p> $5y^2 + 8y - 4 = 0$ </p> <p> $(5y - 2)(y + 2) = 0$ </p> <p> $y = \frac{2}{5}$ or $y = -2$ </p> <p> $x = \frac{3}{5}$ or $x = -3$ </p> <p> $(x ; y) = \left(\frac{3}{5}; \frac{2}{5}\right)$ or $(-3 ; -2)$ </p> <p>OR</p> <p> $3y = 2x$ </p> <p> $x = \frac{3y}{2}$ </p> <p> $\left(\frac{3y}{2}\right)^2 - y^2 + 2\left(\frac{3y}{2}\right) - y = 1$ </p> <p> $\frac{9y^2}{4} - y^2 + 3y - y = 1$ </p> <p> $9y^2 - 4y^2 + 8y = 4$ </p> <p> $5y^2 + 8y - 4 = 0$ </p> <p> $(5y - 2)(y + 2) = 0$ </p> <p> $y = \frac{2}{5}$ or $y = -2$ </p> <p> $x = \frac{3}{5}$ or $x = -3$ </p> <p> $(x ; y) = \left(\frac{3}{5}; \frac{2}{5}\right)$ or $(-3 ; -2)$ </p>	<p style="text-align: right;">(7)</p> <ul style="list-style-type: none"> ✓ simplification of original quadratic ✓ substitution <li style="padding-left: 20px;">$2x = 3y$ ✓ simplification ✓ standard form ✓ factors or substitution into correct formula ✓ y-answers ✓ x-answers <p style="text-align: right;">(7)</p> <ul style="list-style-type: none"> ✓ $x = \frac{3y}{2}$ ✓ substitution ✓ simplification ✓ standard forms ✓ factors or substitution into correct formula ✓ y-answers ✓ x-answers <p style="text-align: right;">(7)</p>
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<p>1.3</p> $\frac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}}$ $= \frac{5^{2007} + 5^{2007} \cdot 5^3}{5^{2008} + 5^{2008} \cdot 5}$ $= \frac{5^{2007} (1 + 5^3)}{5^{2008} (1 + 5)}$ $= \frac{126}{5 \times 6}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4 <p>OR</p> $\frac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}} \quad (\text{divide each term by } 5^{2007})$ $= \frac{5^{2007} + 5^{2007} \cdot 5^3}{5^{2007} \cdot 5 + 5^{2007} \cdot 5^2}$ $= \frac{1 + 5^3}{5 + 5^2}$ $= \frac{126}{30}$ ≈ 4 <p>OR</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 33%; vertical-align: top;"> <p>let $x = 2009$</p> $\frac{5^{x-2} + 5^{x+1}}{5^{x-1} + 5^x}$ $= \frac{5^x (5^{-2} + 5)}{5^x (5^{-1} + 1)}$ $= \frac{1}{25} + 5$ $= \frac{1}{5} + 1$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4 </td> <td style="width: 33%; vertical-align: top; text-align: center;"> <p>OR</p> </td> <td style="width: 33%; vertical-align: top;"> <p>let $x = 2007$</p> $\frac{5^x + 5^{x+3}}{5^{x+1} + 5^{x+2}}$ $= \frac{5^x (1 + 5^3)}{5^x (5 + 5^2)}$ $= \frac{1 + 125}{5 + 25}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4 </td> </tr> <tr> <td style="width: 33%; vertical-align: top;"> <p>let $x = 2010$</p> $\frac{5^{x-3} + 5^x}{5^{x-2} + 5^{x-1}}$ $= \frac{5^x (5^{-3} + 1)}{5^x (5^{-2} + 5^{-1})}$ $= \frac{\frac{1}{125} + 1}{\frac{1}{25} + \frac{1}{5}}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4 </td> <td style="width: 33%;"></td> <td style="width: 33%;"></td> </tr> </table>	<p>let $x = 2009$</p> $\frac{5^{x-2} + 5^{x+1}}{5^{x-1} + 5^x}$ $= \frac{5^x (5^{-2} + 5)}{5^x (5^{-1} + 1)}$ $= \frac{1}{25} + 5$ $= \frac{1}{5} + 1$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4	<p>OR</p>	<p>let $x = 2007$</p> $\frac{5^x + 5^{x+3}}{5^{x+1} + 5^{x+2}}$ $= \frac{5^x (1 + 5^3)}{5^x (5 + 5^2)}$ $= \frac{1 + 125}{5 + 25}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4	<p>let $x = 2010$</p> $\frac{5^{x-3} + 5^x}{5^{x-2} + 5^{x-1}}$ $= \frac{5^x (5^{-3} + 1)}{5^x (5^{-2} + 5^{-1})}$ $= \frac{\frac{1}{125} + 1}{\frac{1}{25} + \frac{1}{5}}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4			<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Note: If the candidate leaves the answer as 4,2 max 2 / 3 marks</p> <p>Answer only of 4,2 0 / 3 marks</p> </div> <p>✓ $\frac{5^{2007} + 5^{2007} \cdot 5^3}{5^{2008} \cdot 5 + 5^{2008} \cdot 5^2}$</p> <p>✓ simplification to $\frac{1+5^3}{5+5^2}$ or $\frac{126}{30}$ or $\frac{21}{5}$</p> <p>✓ answer = 4 (3)</p> <p>✓ $\frac{5^{2007} + 5^{2007} \cdot 5^3}{5^{2007} \cdot 5 + 5^{2007} \cdot 5^2}$</p> <p>✓ simplification to $\frac{1+5^3}{5+5^2}$ or $\frac{126}{30}$ or $\frac{21}{5}$</p> <p>✓ answer = 4</p> <p>✓ $\frac{5^{x-2} + 5^{x+1}}{5^{x-1} + 5^x}$ or $\frac{5^x + 5^{x+3}}{5^{x+1} + 5^{x+2}}$ or $\frac{5^{x-3} + 5^x}{5^{x-2} + 5^{x-1}}$</p> <p>✓ simplification to $\frac{1}{25} + 5$ or $\frac{1}{5} + 1$ or $\frac{1+125}{5+25}$ or $\frac{\frac{1}{125} + 1}{\frac{1}{25} + \frac{1}{5}}$</p> <p>✓ answer = 4 (3)</p> <p style="text-align: right;">[21]</p>
<p>let $x = 2009$</p> $\frac{5^{x-2} + 5^{x+1}}{5^{x-1} + 5^x}$ $= \frac{5^x (5^{-2} + 5)}{5^x (5^{-1} + 1)}$ $= \frac{1}{25} + 5$ $= \frac{1}{5} + 1$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4	<p>OR</p>	<p>let $x = 2007$</p> $\frac{5^x + 5^{x+3}}{5^{x+1} + 5^{x+2}}$ $= \frac{5^x (1 + 5^3)}{5^x (5 + 5^2)}$ $= \frac{1 + 125}{5 + 25}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4					
<p>let $x = 2010$</p> $\frac{5^{x-3} + 5^x}{5^{x-2} + 5^{x-1}}$ $= \frac{5^x (5^{-3} + 1)}{5^x (5^{-2} + 5^{-1})}$ $= \frac{\frac{1}{125} + 1}{\frac{1}{25} + \frac{1}{5}}$ $= \frac{126}{30}$ $= \frac{21}{5}$ $= 4 \frac{1}{5}$ ≈ 4							

QUESTION 2

<p>2.1</p>	$\sum_{n=1}^{20} 3^{n-2}$ $= \frac{1}{3} + 1 + 3 + \dots \text{ to 20 terms}$ $= \frac{1}{3} (3^{20} - 1)$ $= \frac{3^{20} - 1}{3 - 1} \quad ; \quad r = 3; n = 20$ $= \frac{3^{20} - 1}{6}$ $= 581130733,33 \quad \text{OR} \quad 581130733\frac{1}{3} \quad \text{OR} \quad 581130733,3$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If leave only as</p> $\frac{1}{3} + 1 + 3 + 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683$ $+ 59049 + 177147 + 531441 + 1594323 + 4782969$ $+ 14348907 + 43046721 + 129140163 + 387420489$ <p>only, then 2 / 4</p> <p>Note: The 20th term is 387 420 489</p> <p>Answer only: 3 / 4 marks</p> </div>	<p>✓ $a = \frac{1}{3}$</p> <p>✓ $r = 3$</p> <p>✓ $n = 20$</p> <p>✓ answer (4)</p>
<p>2.2.1</p>	$5x ; x^2 ; \frac{x^3}{5} ; \dots$ $r = \frac{x}{5}$ $-1 < \frac{x}{5} < 1$ $-5 < x < 5$ <p>Answer can be written as $x \in (-5 ; 5)$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If $-1 < x < 1$ 1 mark</p> <p>Note: If answer is $-5 \leq x \leq 5$ then 2 / 3</p> </div>	<p>✓ $r = \frac{x}{5}$ or $\frac{x^2}{5x}$</p> <p>✓ $-1 < r < 1$</p> <p>✓ answer (3)</p>
<p>2.2.2</p>	$r = \frac{2}{5} \text{ and } a = 10$ $S_{\infty} = \frac{10}{1 - \frac{2}{5}}$ $= \frac{50}{3} \text{ or } 16,67$	<p>✓ $a = 10$</p> <p>✓ answer (2)</p>

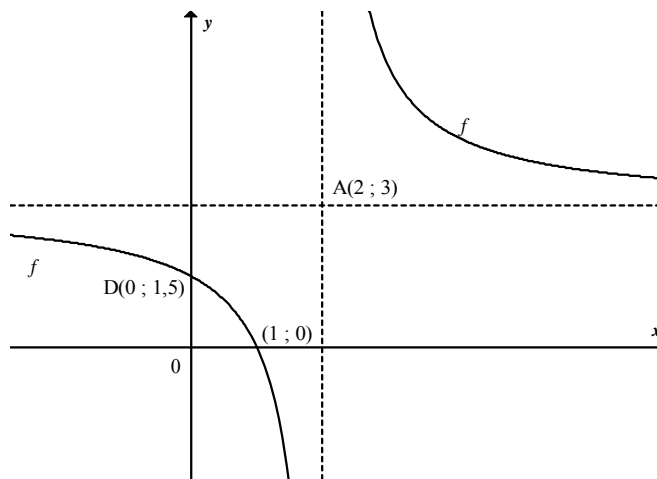
<p>2.3.1</p>	$T_n = 20 + 3(n - 1)$ $101 = 20 + (n - 1)3$ $84 = 3n$ $n = 28$ <p>OR</p> $T_n = 3n + 17$ $101 = 3n + 17$ $84 = 3n$ $n = 28$	<p>Note: If $n = -\frac{17}{3}$ Then 1 / 2 marks</p> <p>Answer only: Full marks</p>		<p>✓ 101 = 20 + 3(n - 1) or 101 = 3n + 17 ✓ answer (2)</p> <p>✓ substitution ✓ answer (2)</p>
<p>2.3.2</p>	<p>23 + 29 + ... to 14 terms</p> $= \frac{14}{2}[2(23) + (14 - 1)6] \quad \text{OR} \quad \frac{14}{2}[23 + 101]$ $= 868$ <p>OR</p> <p>Even numbers = 20 ; 26 ; ... ; 98</p> $T_n = 6n + 14 \qquad T_n = 20 + (n - 1)6$ $98 = 6n + 14 \qquad \text{OR} \qquad 98 = 20 + (n - 1)6$ $84 = 6n \qquad \qquad \qquad 84 = 6n$ $14 = n \qquad \qquad \qquad 14 = n$ $S_{\text{remaining}} = \frac{28}{2}[2(20) + (27)(3)] - \frac{14}{2}[2(20) + (13)(6)]$ $= 14(121) - 7(118)$ $= 1694 - 826$ $= 868$ <p>OR</p> <p>Sequence is 20; 23; 26; 29; 32; 35; 38; 41; 44; 47; 50; 53; 56; 59; 62; 65; 68; 71; 74; 77; 80; 83; 86; 89; 92; 95; 98; 101</p> <p>Sum of odd numbers = 23 + 29 + 35 + 41 + 47 + 53 + 59 + 65 + 71 + 77 + 83 + 89 + 95 + 101 = 868</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Note: If “to 14 terms” is left out, do not penalise</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Note: If incorrect value for n, max 4 / 6</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Note: If incorrect formula, max 2 / 6</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If the candidate only works out the even numbers i.e. 826, then 3 / 6 marks</p> <p>If only 1694 max 1 / 6 marks</p> </div>		<p>✓ 23 + 29 + ... ✓ $a = 23$ ✓ $n = 14$</p> <p>✓ $d = 6$ or $l = 101$ ✓ substitution into correct formula ✓ answer (6)</p> <p>OR</p> <p>✓ $98 = 6n + 14$ or $98 = 20 + (n - 1)$ ✓ $14 = n$</p> <p>✓ substitution into correct formula ✓ 1694</p> <p>✓ 826 ✓ answer (6)</p> <p>Full marks (6)</p> <p>[17]</p>

QUESTION 3

<p>3.1</p>	<p>First difference : 5; $x - 9$; $37 - x$ Second difference : $x - 14$; $- 2x + 46$ $x - 14 = 46 - 2x$ $3x = 60$ $x = 20$</p> <p>OR</p> <p>$(x - 9) + (x - 14) = 37 - x$ $2x - 23 = 37 - x$ $3x = 60$ $x = 20$</p> <p>OR</p> <p>$(x - 9) - 5 = (37 - x) - (x - 9)$ $x - 14 = -2x + 46$ $3x = 60$ $x = 20$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Answer only: Full Marks</p> </div>	<p>✓ first differences 5; $x - 9$; $37 - x$ ✓ seconds difference ✓ answer (3)</p> <p>✓ equating ✓ manipulation ✓ answer (3)</p> <p>✓ first differences 5; $x - 9$; $37 - x$ ✓ equating ✓ answer (3)</p>
<p>3.2</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If x is incorrect in 3.1 then max 2 / 4 marks</p> </div> <p>$2a = 6$ $a = 3$ $T_n = 3n^2 + bn + c$ $3 + b + c = 4 \dots T_1$ $b + c = 1$ $12 + 2b + c = 9 \dots T_2$ $2b + c = -3$ $\therefore 9 + b = 5$ $b = -4$ and $c = 4 - (-1) = 5$ $\therefore T_n = 3n^2 - 4n + 5$</p> <p>OR</p>	<p>✓ $a = 3$ ✓ $T_n = 3n^2 + bn + c$</p> <p>✓ $b = -4$ ✓ $c = 5$ (4)</p>

	$2a = 6$ $a = 3$ $T_0 = 5$ $c = 5$ $T_n = 3n^2 + bn + 5$ $4 = 3(1)^2 + b + 5$ $b = -4$ $T_n = 3n^2 - 4n + 5$ <p>OR</p> $a + b + c = 4 \quad \dots \text{i}$ $4a + 2b + c = 9 \quad \dots \text{ii}$ $16a + 4b + c = 37 \quad \dots \text{iii}$ $3a + b = 5$ $12a + 2b = 28$ $6a + b = 14$ $3a = 9$ $a = 3$ $b = -4$ $c = 5$ $T_n = 3n^2 - 4n + 5$	$2a = 6$ $a = 3$ $3a + b = 5$ $b = -4$ $a + b + c = 4$ $3 - 4 + c = 4$ $c = 5$ $T_n = 3n^2 - 4n + 5$ <p>OR</p> $T_n = 4 + (n-1)5 + \frac{6(n-1)(n-2)}{2}$ $= 4 + 5n - 5 + 3n^2 - 9n + 6$ $= 3n^2 - 4n + 5$	<p>OR</p> <p>✓ $a = 3$</p> <p>✓ $c = 5$</p> <p>✓ method</p> <p>✓ $b = -4$ (4)</p> <p>✓ $a = 3$</p> <p>✓ $c = 5$</p> <p>✓ method</p> <p>✓ $b = -4$ (4)</p> <p>[7]</p>
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QUESTION 4



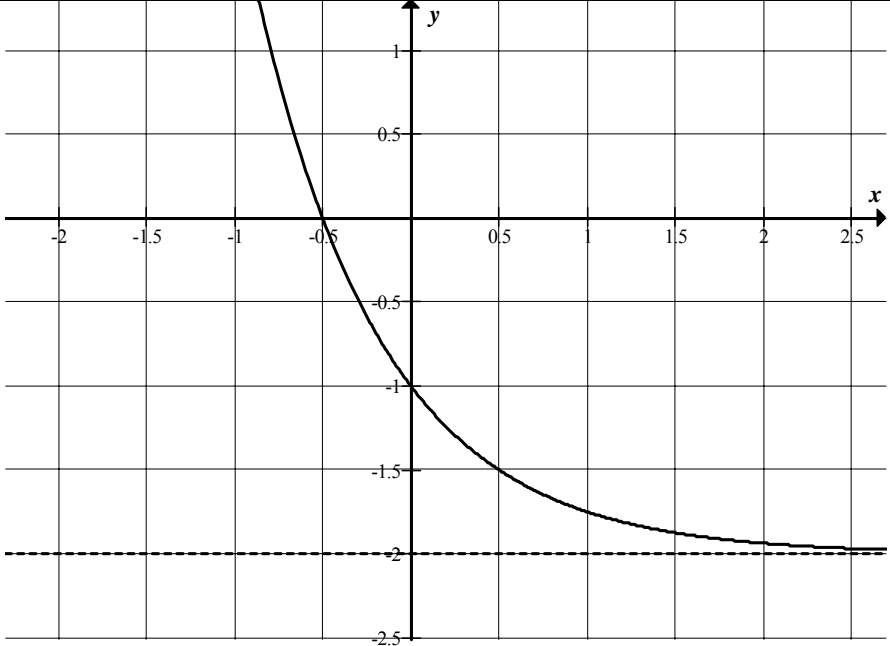
<p>4.1</p>	<p>$x = 2$ $y = 3$</p> <p style="text-align: center;">OR</p> <p>x-asymptote = 2 y-asymptote = 3</p> <p>If $x = p ; y = q$ then 1 mark</p> <p>Note: If the candidate just writes down the number 2 or 3 or just coordinates (2 ; 3), then no marks</p>	<p>✓ answer ✓ answer (2)</p>
<p>4.2</p>	<p>$f(x) = \frac{a}{x-2} + 3$</p> <p>$0 = \frac{a}{1-2} + 3$</p> <p>$0 = -a + 3$</p> <p>$a = 3$</p> <p>$f(x) = \frac{3}{x-2} + 3$</p> <p>OR</p> <p>$y = \frac{a}{x-2} + 3$</p> <p>$y - 3 = \frac{a}{x-2}$</p> <p>$(x-2)(y-3) = a$</p> <p>But (1;0) lies on the graph</p> <p>$\therefore (-1)(-3) = a = 3$</p> <p>$\therefore (x-2)(y-3) = 3$</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>If the asymptotes are swapped in 4.1, then</p> <p>$f(x) = \frac{a}{x-3} + 2$</p> <p>$0 = \frac{a}{1-3} + 2$</p> <p>$a = 4$</p> <p>$f(x) = \frac{4}{x-3} + 2$</p> </div>	<p>✓ subs in of asymptotes ✓ subs in (1 ; 0) ✓ answer (3) ✓ equation ✓ subs in (1 ; 0) ✓ answer (3)</p>
<p>4.3</p>	<p>When $x = 0, y = \frac{3}{0-2} + 3$</p> <p style="text-align: center;">$= \frac{3}{2}$</p> <p>$D\left(0; \frac{3}{2}\right)$</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>If asymptotes swapped:</p> <p>$x = 0$</p> <p>$y = \frac{4}{0-3} + 2$</p> <p>$y = \frac{2}{3}$</p> <p>$D\left(0; \frac{2}{3}\right)$</p> </div>	<p>✓ $x = 0$ ✓ $y = \frac{3}{2}$ (2)</p>

<p>4.4</p>	$m_{AD} = \frac{3 - 1,5}{2 - 0}$ $= \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$ <p>OR</p> $4y = 3x + 6$ <p>OR</p> $y = mx + \frac{3}{2}$ $3 = m(2) + \frac{3}{2}$ $m = \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>If asymptotes swapped:</p> $m_{AD} = \frac{3 - \frac{2}{3}}{2 - 0}$ $= \frac{7}{3} \times \frac{1}{2}$ $= \frac{7}{6}$ $y = \frac{7}{6}x + \frac{2}{3}$ </div>	<p>✓ substitution into gradient</p> <p>✓ $\frac{3}{4}$</p> <p>✓ answer (3)</p> <p>✓ substitution of point (2 ; 3) and $c = \frac{3}{2}$</p> <p>✓ $\frac{3}{4}$</p> <p>✓ answer (3)</p>
<p>4.5</p>	$\frac{p + 0}{2} = 2$ $p = 4$ $\frac{q + \frac{3}{2}}{2} = 3$ $q = 4\frac{1}{2}$ <p>Other point of intersection is $\left(4; 4\frac{1}{2}\right)$</p> <p>OR</p> <p>By symmetry the rule to calculate the point of intersection is $(x; y) \rightarrow \left(x + 2; y + \frac{3}{2}\right)$</p> <p>Other point of intersection is $\left(2 + 2; 3 + \frac{3}{2}\right)$</p> $= \left(4; 4\frac{1}{2}\right)$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>Answer only: Full Marks</p> </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>To help with applying CA the y-coordinate will be $3 + (3 - y)$</p> </div>	<p>✓ $\frac{p + 0}{2} = 2$</p> <p>✓ $\frac{q + \frac{3}{2}}{2} = 3$</p> <p>✓ $x = 4$</p> <p>✓ $y = 4\frac{1}{2}$ (4)</p> <p>✓✓ x-answer</p> <p>✓✓ y-answer (4)</p>

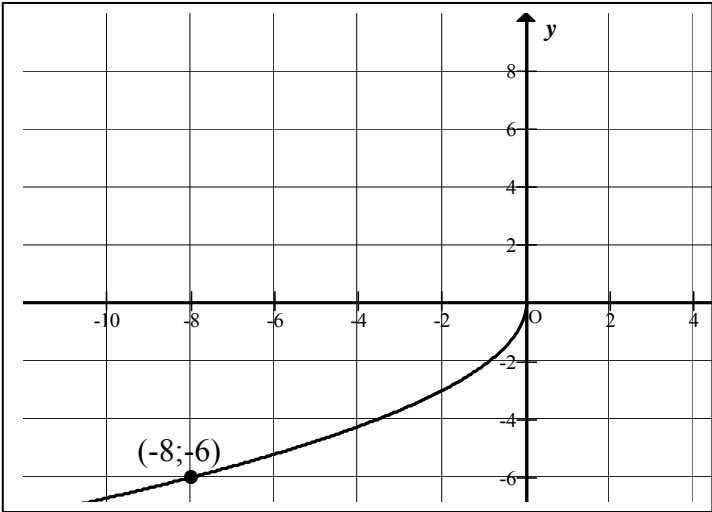
	<p>OR</p> $\frac{3}{4}x + \frac{3}{2} = \frac{3}{x-2} + 3$ $3x(x-2) + 6(x-2) = 12 + 12(x-2)$ $3x^2 - 6x + 6x - 12 = 12 + 12x - 24$ $3x^2 - 12x = 0$ $3x(x-4) = 0$ $x = 0 \text{ and } x = 4$ <p>Other point of intersection is $\left(4; 4\frac{1}{2}\right)$</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: 80%;"> <p>Note: If the candidate does not select the x-value greater than 2 i.e. a realistic answer, max 3 / 4 marks</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: 80%;"> <p>If asymptotes swapped:</p> $\frac{7}{6}x + \frac{2}{3} = \frac{4}{x-3} + 2$ $7x(x-3) + 4(x-3) = 4(6) + 2(6)(x-3)$ $7x^2 - 29x = 0$ $x(7x-29) = 0$ $x = 0 \text{ or } x = \frac{29}{7}$ <p>Other point of intersection is $\left(\frac{29}{7}; \frac{11}{2}\right)$</p> </div>	<p>✓ equating</p> <p>✓ standard form</p> <p>✓ x-values</p> <p>✓ y-value</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">[14]</p>
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QUESTION 5

<p>5.1</p>	<p>$f(x) = 4^{-x} - 2$</p> <p>y-intercept: $x = 0; y = 4^0 - 2 = -1 ; (0 ; -1)$</p> <p>$x$-intercept:</p> $4^{-x} - 2 = 0$ $4^{-x} = 2$ $\log 4^{-x} = \log 2$ $-x = \frac{\log 2}{\log 4} \quad \text{OR} \quad -x = \frac{\log 2}{2 \log 2}$ $x = -\frac{1}{2}$ <p>x-intercept is $\left(-\frac{1}{2}; 0\right)$</p>	<p>✓✓ y-intercept</p> <p>✓✓ x-intercept</p> <p style="text-align: right;">(4)</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: No penalty if the answer is not left as a coordinate.</p> </div>
<p>5.2</p>	<p>$y = -2$</p>	<p>✓ equation</p> <p style="text-align: right;">(1)</p>

<p>5.3</p>		<ul style="list-style-type: none"> ✓ asymptote ✓ y-intercept or x-intercept ✓ shape (decreasing) <p style="text-align: right;">(3)</p>
<p>5.4</p>	<p>$g(x) = 4^{-x} - 2 + 2$ $g(x) = 4^{-x}$</p> <p>OR $g(x) = \left(\frac{1}{4}\right)^x$</p> <p>OR $g(x) = 2^{-2x}$</p> <p>OR $g(x) = \left(\frac{1}{2}\right)^{2x}$</p>	<p>✓ equation</p> <p style="text-align: right;">(1)</p>
<p>5.5</p>	<p>$4^{-x} - 2 = 3$ $4^{-x} = 5$ $-x \log 4 = \log 5$ $x = -\frac{\log 5}{\log 4}$ OR $x = -\log_4 5$ OR $x = \log_{\frac{1}{4}} 5$ OR $x = \log_4 \frac{1}{5}$</p> <p>OR $x = -1,16$ OR $x = \frac{\log 5}{\log \frac{1}{4}}$ OR $x = \frac{\log \frac{1}{5}}{\log 4}$</p>	<ul style="list-style-type: none"> ✓ $4^{-x} = 5$ ✓ $-x \log 4 = \log 5$ <p>✓ answer</p> <p style="text-align: right;">(3) [12]</p>

QUESTION 6

<p>6.1</p>	$f(x) = ax^2$ $-8 = a(-6)^2$ $-8 = 36a$ $a = -\frac{8}{36}$ <p>OR</p> $a = -\frac{2}{9}$	<p>✓ substitution</p> <p>✓ answer (2)</p>
<p>6.2</p>	$f(x) : y = -\frac{2}{9}x^2$ $x = -\frac{2}{9}y^2$ $9x = -2y^2$ $-\frac{9x}{2} = y^2$ $y = \pm\sqrt{\frac{-9x}{2}}, \text{ since } y \leq 0$ $y = -\sqrt{\frac{-9x}{2}} \text{ OR } y = -3\sqrt{\frac{-x}{2}}$ <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Note: If candidate does not substitute the value of a the answer is $y = -\sqrt{\frac{x}{a}}$ then 2 / 3 marks</p> </div>	<p>✓ swop x and y</p> <p>✓ $y^2 = -\frac{9x}{2}$ or $y = \pm\sqrt{\frac{-9x}{2}}$</p> <p>✓ $y = -\sqrt{\frac{-9x}{2}}$ (3)</p>
<p>6.3</p>	<p>$y \leq 0$</p> <p>OR</p> <p>$y \in (-\infty ; 0]$</p>	<p>✓ answer (1)</p>
<p>6.4</p>		<p>✓ shape (third quadrant) (concave upward)</p> <p>✓ Any point other than (0 ; 0) that lies on the graph</p> <p>Point corresponding from original graph will be (- 8 ; - 6) (2)</p>

<p>6.5</p>	$y = -f^{-1}(x)$ $= \sqrt{\frac{-9x}{2}}$ <p>OR</p> $y = -\frac{2}{9}x^2$ <p>Reflection in $y = x$: $x = -\frac{2}{9}y^2$</p> $-\frac{9}{2}x = y^2$ $y = -\sqrt{-\frac{9x}{2}}$ <p>Reflection about y-axis: $y = \sqrt{-\frac{9x}{2}}$</p>	$y = -f^{-1}(x)$ $= 3\sqrt{\frac{-x}{2}}$ <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Note: If candidate has $(x ; y) \rightarrow (y ; -x)$ then 2 / 3 marks</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Note: If candidate does not substitute the value of a the answer is $y = \sqrt{\frac{x}{a}}$ then full marks</p> </div>	<p>✓✓ $-f^{-1}(x)$</p> <p>✓ answer (3)</p> <p>✓ $x = -\frac{2}{9}y^2$</p> <p>✓ $y = -\sqrt{-\frac{9x}{2}}$</p> <p>✓ $y = \sqrt{-\frac{9x}{2}}$</p> <p>(3) [11]</p>
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QUESTION 7

<p>7.1</p>	$A = P(1+i)^n$ $2P = P\left(1 + \frac{r}{4}\right)^{6 \times 4}$ $2 = \left(1 + \frac{r}{4}\right)^{24}$ $1 + \frac{r}{4} = 2^{\frac{1}{24}}$ $r = 4\left(2^{\frac{1}{24}} - 1\right)$ $r = 4\left(2^{\frac{1}{24}}\right) - 4$ $r = 0,1172 \dots$ <p>rate = 11,72% p.a. compounded quarterly</p> <p>OR</p> $A = P(1+i)^n$ $2P = P\left(1 + \frac{r}{400}\right)^{6 \times 4}$ $2 = \left(1 + \frac{r}{400}\right)^{24}$ $1 + \frac{r}{400} = 2^{\frac{1}{24}}$ $r = 400\left(2^{\frac{1}{24}} - 1\right)$ $r = 400\left(2^{\frac{1}{24}}\right) - 400$ $r = 11,72\% \text{ p.a.}$	<p>✓ 2P</p> <p>✓ $\frac{r}{4}$ and 24</p> <p>✓ $1 + \frac{r}{4} = 2^{\frac{1}{24}}$</p> <p>✓ $r = 4\left(2^{\frac{1}{24}}\right) - 4$</p> <p>✓ answer (5)</p> <p>✓ 2P</p> <p>✓ $\frac{r}{400}$ and 24</p> <p>✓ $1 + \frac{r}{400} = 2^{\frac{1}{24}}$</p> <p>✓ $r = 400\left(2^{\frac{1}{24}}\right) - 400$</p> <p>✓ answer (5)</p>
<p>7.2.1</p>	$A = 10000\left(1 + \frac{0,095}{12}\right)^5$ $= R 10 402,15$	<p>✓ substitution in correct formula</p> <p>✓ answer (2)</p>

7.2.2	$10402,15 = \frac{450 \left[1 - \left(1 + \frac{0,095}{12} \right)^{-n} \right]}{\frac{0,095}{12}}$ $0,183000787 = 1 - \left(1 + \frac{0,095}{12} \right)^{-n}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> <p>Note: Incorrect Formula No marks</p> </div> $\left(1 + \frac{0,095}{12} \right)^{-n} = 0,816999213$ $\log \left(1 + \frac{0,095}{12} \right)^{-n} = \log 0,816999213$ $-n \log \left(1 + \frac{0,095}{12} \right) = \log 0,816999213 \dots$ $n = 25,63151282 \dots$ $n = 25,63 \text{ months}$ $n = 26$ <p>Accept: $n = 31$ (because of first 5 months)</p> <p>OR</p> $10402,15 \left(1 + \frac{0,095}{12} \right)^n = \frac{450 \left[\left(1 + \frac{0,095}{12} \right)^n - 1 \right]}{\frac{0,095}{12}}$ $10402,15 \left(1 + \frac{0,095}{12} \right)^n = 56842,10526 \left[\left(1 + \frac{0,095}{12} \right)^n - 1 \right]$ $56842,10526 = 46439,95526 \left(1 + \frac{0,095}{12} \right)^n$ $\log 1,223991387 = n \log \left(1 + \frac{0,095}{12} \right)$ $n = \frac{\log 1,223991387}{\log \left(1 + \frac{0,095}{12} \right)}$ $n = 25,63 \text{ months}$ $n = 26$ <p>Accept: $n = 31$ (because of first 5 months)</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Note: If the Present value of R 10 000 is used, then $n = 25,53$ months is obtained. Max 3 / 4 marks.</p> </div>	<p>✓ 10 402,15 ✓ substitution into present value formula</p> <p>✓ application of logs</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p> <p>✓ 10 402,15 ✓ substitution into future value formula</p> <p>✓ application of logs</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
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<p>7.2.3</p> <p>Balance outstanding after 25 months</p> $= 10402,15 \left(1 + \frac{0,095}{12}\right)^{25} - \frac{450 \left[\left(1 + \frac{0,095}{12}\right)^{25} - 1 \right]}{\frac{0,095}{12}}$ <p>= R 282,36</p> <p>OR</p> <p>Balance Outstanding after 25 months</p> $= 10000 \left(1 + \frac{0,095}{12}\right)^{30} - \frac{450 \left[\left(1 + \frac{0,095}{12}\right)^{25} - 1 \right]}{\frac{0,095}{12}}$ <p>= R 282,36</p> <p>OR</p> <p>$n = 25,6315128204\dots - 25$ $= 0,6315128204 \dots$</p> <p>Balance Outstanding after 25 months</p> $= \frac{450 \left[1 - \left(1 + \frac{0,095}{12}\right)^{-0,631512804} \right]}{\frac{0,095}{12}}$ <p>= R 282,36</p> <p>OR</p> <p>Present value at beginning of 25 months</p> $= 10402,15 - \frac{450 \left[1 - \left(1 + \frac{0,095}{12}\right)^{-25} \right]}{\frac{0,095}{12}}$ <p>= R 231,84</p> <p>Balance Outstanding</p> $= 231,84 \left(1 + \frac{0,095}{12}\right)^{25}$ <p>= R 282,36</p>	<p>✓ correct formula ✓ substitution into</p> $\frac{450 \left[\left(1 + \frac{0,095}{12}\right)^{25} - 1 \right]}{\frac{0,095}{12}}$ <p>✓ answer (3)</p> <p>✓ correct formula ✓</p> $\frac{450 \left[\left(1 + \frac{0,095}{12}\right)^{25} - 1 \right]}{\frac{0,095}{12}}$ <p>✓ answer (3)</p> <p>✓ correct formula ✓ substitution into</p> $\frac{450 \left[1 - \left(1 + \frac{0,095}{12}\right)^{-0,631512804} \right]}{\frac{0,095}{12}}$ <p>✓ answer (3)</p> <p>✓ correct formula ✓ substitution into</p> $\frac{450 \left[1 - \left(1 + \frac{0,095}{12}\right)^{-25} \right]}{\frac{0,095}{12}}$ <p>✓ answer (3)</p> <p style="text-align: right;">[14]</p>
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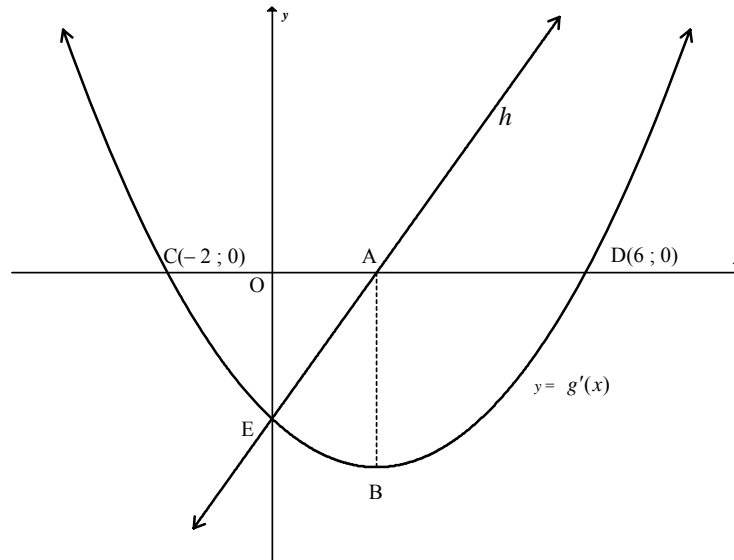
Note: Accept
 If a candidate uses
 - 0,63, the final
 answer is R 281,68

QUESTION 8

<p>8.1</p>	$g(x) = x^2 - 5$ $g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 5 - (x^2 - 5)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $= \lim_{h \rightarrow 0} (2x + h)$ $= 2x$ <p>OR</p> $g(x) = x^2 - 5$ $g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 5 - (x^2 - 5)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h+x)(x+h-x)}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$ $= \lim_{h \rightarrow 0} (2x+h)$ $= 2x$	<p>Note: If the notation is incorrect, penalty 1 mark</p> <p>If candidate subtracts and gets $x^2 + 2xh + h^2 - 5 - x^2 - 5$ in the numerator and then candidate corrects themselves, max 2 / 5</p> <p>Answer only: 0 / 5</p>	<ul style="list-style-type: none"> ✓ formula ✓ substitution ✓ expansion ✓ 2x + h ✓ answer <p style="text-align: right;">(5)</p>
<p>8.2</p>	$y = \frac{x^6}{2} + 4\sqrt{x}$ $y = \frac{1}{2}x^6 + 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 3x^5 + 2x^{-\frac{1}{2}}$	<p>Note: If $\frac{dy}{dx}$ or y' is left out, penalty 1 mark</p> <p>If a candidate shows evidence of how to differentiate from an incorrect function which involves breakdown, then max 1 / 3</p>	<ul style="list-style-type: none"> ✓ $+4x^{\frac{1}{2}}$ ✓ $3x^5$ ✓ $2x^{-\frac{1}{2}}$ <p style="text-align: right;">(3)</p>

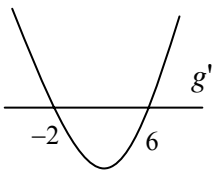
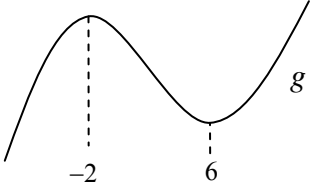
<p>8.3</p> $g(x) = ax^2 + \frac{b}{x}$ $g(x) = ax^2 + bx^{-1}$ $g'(x) = 2ax - bx^{-2}$ $0 = 2a(4) - \frac{b}{(4)^2}$ $8a = \frac{b}{16}$ $b = 128a$ $96 = a(4)^2 + \frac{b}{4}$ $96 = 16a + \frac{1}{4}(128a)$ $96 = 48a$ $a = 2$ $b = 256$ <p>OR</p> $g'(x) = 2ax - \frac{b}{x^2}$ $g'(4) = 8a - \frac{b}{16} = 0$ $g(4) = 16a + \frac{b}{4} = 96$ $32a - \frac{b}{4} = 0$ $48a = 96$ $a = 2$ $b = 256$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>Note: In the equation $g'(x) = 0 ; = 0$ must be shown in the equation.</p> </div>	$\checkmark g'(x) = 2ax - bx^{-2}$ $\checkmark 0 = g'(x)$ $\checkmark 2a(4) - \frac{b}{(4)^2}$ $\checkmark \text{subs } (4 ; 96)$ $\checkmark a = 2$ $\checkmark b = 256$ <p style="text-align: right;">(6)</p> $\checkmark g'(x) = 2ax - \frac{b}{x^2}$ $\checkmark g'(4) = 8a - \frac{b}{16}$ $\checkmark g'(x) = 0$ $\checkmark g(4) = 16a + \frac{b}{4} = 96$ $\checkmark a = 2$ $\checkmark b = 256$ <p style="text-align: right;">(6) [14]</p>
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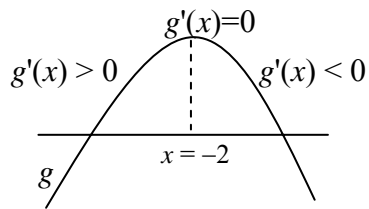
QUESTION 9



<p>9.1</p>	<p>The y-intercept of g is E(0 ; -4)</p> <p>OR $x = 0$ and $y = -4$</p>	<p>✓ answer (1)</p>
<p>9.2</p>	<p>$y = a(x + 2)(x - 6)$ $-4 = a(0 + 2)(0 - 6)$ $-4 = -12a$ $a = \frac{1}{3}$ $y = \frac{1}{3}(x + 2)(x - 6)$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p> <p>OR $g'(0) = -4 = c$ $g'(x) = ax^2 + bx - 4$ $g'(-2) = 0$ $4a - 2b - 4 = 0$ $b = 2a - 2$ $g''(2) = 0$ $2a(2) + b = 0$ $b = -4a$ $2a - 2 = -4a$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p>	<p>✓ setting up of equation ✓ subs (0 ; -4)</p> <p>✓ $a = \frac{1}{3}$</p> <p>✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)</p> <p>✓ substitution $x = -2$ and $g'(x) = 0$</p> <p>✓ $g''(2) = 0$</p> <p>✓ $a = \frac{1}{3}$</p> <p>✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)</p>

<p>OR $c = -4$ $4a - 2b - 4 = 0$ $36a + 6b - 4 = 0$ $48a - 16 = 0$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p> <p>OR $y = a(x + 2)(x - 6)$ $= a(x^2 - 4x - 12)$ $= ax^2 - 4ax - 12a$ $-12a = -4$ $a = \frac{1}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p> <p>OR $\frac{dy}{dx} = 2ax + b$ $0 = 2a(2) + b$ $b = -4a$</p> <p>EITHER subs (6 ; 0) $0 = 36a + 6b - 4$ $4 = 36a + 6b$ $2 = 18a + 3b$ $2 = 18a + 3(-4a)$ $2 = 6a$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p> <p>OR $0 = 4a - 2b - 4$ $0 = 4a - 2(-4a) - 4$ $12a = 4$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$</p>	<p>✓ setting up of equation ✓ simultaneous equation</p> <p>✓ $a = \frac{1}{3}$</p> <p>✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)</p> <p>✓ setting up of equation ✓ $ax^2 - 4ax - 12a$</p> <p>✓ $a = \frac{1}{3}$</p> <p>✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)</p> <p>✓ $b = -4a$</p> <p>✓ simultaneous equation ✓ $a = \frac{1}{3}$</p> <p>✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)</p>
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<p>9.3</p>	<p>At turning point $g'(x) = 0$ $x = -2$ and $x = 6$</p>	<p>Answer only: Full marks</p> <p>If only 1 value given, max 1 / 2</p>	<p>✓ $g'(x) = 0$ ✓ $x = 6$ and $x = -2$ (2)</p>
<p>9.4</p>	<p>$x = \frac{-2+6}{2}$ $x = 2$</p> <p>OR</p> <p>x-value of point of inflection of g is at A. $g''(x) = 0$ $\frac{2x}{3} - \frac{4}{3} = 0$ $2x - 4 = 0$ $2x = 4$ $x = 2$</p> <p>OR</p> <p>$x = -\frac{b}{2a}$ $x = \frac{\frac{4}{3}}{2(\frac{1}{3})}$ $x = 2$</p>	<p>Note: Answer only Full marks</p>	<p>✓ $x = \frac{-2+6}{2}$ ✓ answer (2)</p> <p>✓ $2x - 4 = 0$ ✓ answer (2)</p> <p>✓ $x = \frac{\frac{4}{3}}{2(\frac{1}{3})}$ ✓ answer (2)</p> <p>✓ $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$ ✓ answer (2)</p>
<p>9.5</p>	<p>$g'(x) > 0$ for $x < -2$, so g is increasing for $x < -2$. $g'(x) < 0$ for $x > -2$, so g is decreasing for $x > -2$. $\therefore g$ has a local maximum at $x = -2$ because the graph is increasing followed by decreasing</p> <p>OR</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>$\therefore g$ has a local maximum at $x = -2$</p> <p>OR</p>		<p>✓ $g'(x) > 0$ ✓ g is incr for $x < -2$ ✓ g is decr for $x > -2$ (3)</p> <p>✓ $g'(x) > 0$ for $x < -2$ ✓ $g'(x) < 0$ for $x > -2$ ✓ max at $x = -2$ (3)</p>

**OR**

$$g'(-2) = 0$$

$g''(-2) < 0$ so graph is concave down at $x = -2$, so g has a local maximum

- ✓ $g'(x) > 0$ for $x < -2$
- ✓ $g'(x) < 0$ for $x > -2$
- ✓ max at $x = -2$

(3)

- ✓ $g'(-2) = 0$
- ✓ $g''(-2) < 0$
- ✓ max at $x = -2$

(3)

[12]

QUESTION 10

<p>10.1</p>	$V = \pi r^2 h + 2 \times \frac{1}{2} \times \frac{4}{3} \pi r^3$ $V = \pi r^2 h + \frac{4}{3} \pi r^3$ $\frac{\pi}{6} = \pi r^2 h + \frac{4}{3} \pi r^3$ $\pi r^2 h = \frac{\pi}{6} - \frac{4}{3} \pi r^3$ $h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$ $h = \frac{1}{6r^2} - \frac{4r}{3}$	<p>✓ volume equation ✓ substitution of $\frac{\pi}{6}$ ✓ $h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{\pi r^2}$</p> <p>(3)</p>
<p>10.2</p>	$S = 2 \times 2\pi r^2 + 2\pi rh$ $S = 4\pi r^2 + 2\pi rh$ $S = 4\pi r^2 + 2\pi r \left(\frac{1}{6r^2} - \frac{4r}{3} \right)$ $S = 4\pi r^2 + \frac{\pi}{3r} - \frac{8\pi r^2}{3}$ $= \frac{4}{3} \pi r^2 + \frac{\pi}{3r}$	<p>✓ surface area equation ✓ substitution of h ✓ simplification</p> <p>(3)</p>
<p>10.3</p>	$S = \frac{4}{3} \pi r^2 + \frac{\pi}{3} r^{-1}$ $\frac{dS}{dr} = \frac{8\pi r}{3} - \frac{\pi}{3r^2} = 0$ $8r = \frac{1}{r^2}$ $8r^3 = 1$ $r = \frac{1}{2}$ <p>Then $S = \frac{4}{3} \pi \left(\frac{1}{2} \right)^2 + \frac{\pi}{3} (2)$</p> <p>$S = \pi$ square metres $= 3,14$ square metres</p>	<p>✓ $\frac{\pi}{3} r^{-1}$ ✓ $\frac{dS}{dr} = \frac{\pi}{3} \left(8r - \frac{1}{r^2} \right)$ or $\frac{dS}{dr} = \frac{\pi}{3} (8r - r^{-2})$ ✓ $\frac{dS}{dr} = 0$ ✓ $8r = \frac{1}{r^2}$ ✓ $r = \frac{1}{2}$ ✓ $S = \pi$</p> <p>(6) [12]</p>

QUESTION 11

<p>11.1</p>	<p>$x, y \in N_0$</p> <p>$x + 2y \leq 28$ or $y \leq -\frac{x}{2} + 14$</p> <p>$3x + y \leq 24$ or $y \leq -3x + 24$</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Note: If inequality signs incorrect or equal signs used: max 3 / 4 marks</p> </div>	<p>✓✓ First inequality ✓✓ Second inequality (4)</p>
<p>11.2</p>		<p>✓ graph of $x + 2y \leq 28$ ✓ graph of $3x + y \leq 24$ ✓ feasible region (quadrilate ral) (3)</p>
<p>11.3.1</p>	<p>8</p>	<p>✓ answer (1)</p>
<p>11.3.2</p>	<p>14</p>	<p>✓ answer (1)</p>
<p>11.4</p>	<p>Maximise $x + y$ Use search line with gradient $- 1$</p> <p>4 Type A 12 Type B</p>	<p>✓ 4 Type A ✓ 12 Type B (2)</p>

11.5	$x \geq y$ $y \leq x$ New Feasible region (triangle) in diagram Maximise $x + y$. Maximum at (6 ; 6) Answer: $6 + 6 = 12$ braai stands Machine Time = $x + 2y$ = $6 + 2 \times 6$ = $6 + 12$ = 18 hours	Note: Answer only of machine time 18 hours and braai stands 12 Full marks	$\checkmark y \leq x$ $\checkmark (6 ; 6)$ $\checkmark \checkmark 12$ $\checkmark 18$ hours (5) [16]
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TOTAL: 150



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2010

MEMORANDUM

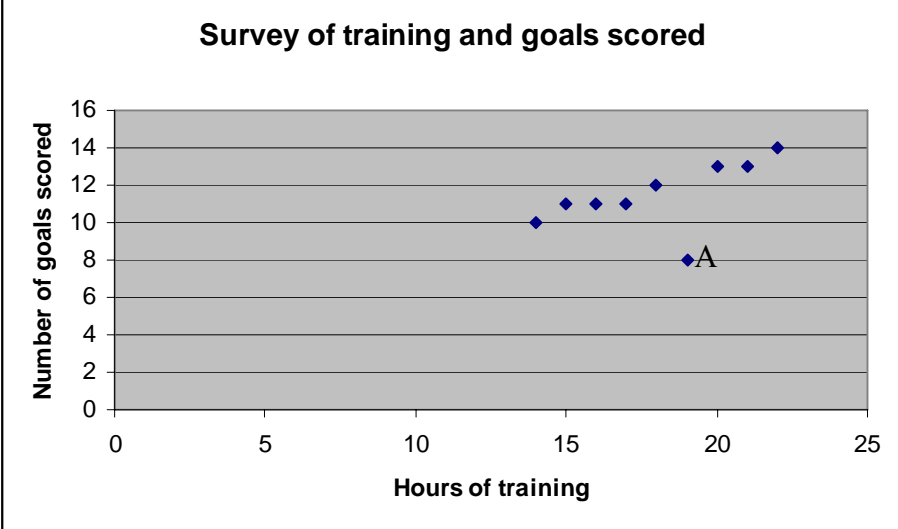
MARKS: 150

This memorandum consists of 14 pages.

QUESTION 1

1.1	Range = $26 - 4 = 22$	✓ maximum and minimum values ✓ answer ANSWER ONLY: Full Marks (2)
1.2	Mean $= \frac{4 + 5 + 8 + 13 + 19 + 22 + 25 + 26 + 23 + 17 + 14 + 7}{12}$ $= \frac{183}{12}$ $= 15,25$	✓ method ✓ 183 ✓ answer (3)
1.3	Standard deviation = 7,6 (7,59522.....)	✓✓ answer (2)
1.4.1	Increase in mean = $\frac{(3 \times 5) + (9 \times 1)}{12}$ $= 2^\circ\text{C per month.}$	✓✓ answer (2)
1.4.2	The maximum value increases by 1°C and the minimum value increases by 5°C . This implies that the range of the range of the data will now decrease. This will result in the standard deviation getting smaller. (new SD = 6,27.....)	✓ decrease in range ✓ decrease in standard deviation (2) [11]

QUESTION 2

2.1.1	<p style="text-align: center;">Survey of training and goals scored</p> 	<p>✓✓✓ plotting the points</p> <p>All 9 point correct – 3 marks 5 or 7 points correct – 2 marks 1 or 2 points correct – 1 mark 0 points correct – 0 marks</p> <p style="text-align: right;">(3)</p>
2.1.2	A(indicated on the graph)	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
2.1.3	8 Goals	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p>
2.2	<p>Let the mean time for all 560 learners be x. Then the mean time for the learners living in neighbourhood C is also x.</p> $x = \frac{(135 \times 24) + (225 \times 32) + (200 \times x)}{560}$ $560x = 3240 + 7200 + 200x$ $360x = 10440$ $x = 29$	<p>✓ equal mean times</p> <p>✓ mean \times number</p> <p>✓ simplification</p> <p>✓ answer</p> <p style="text-align: right;">(4) [10]</p>

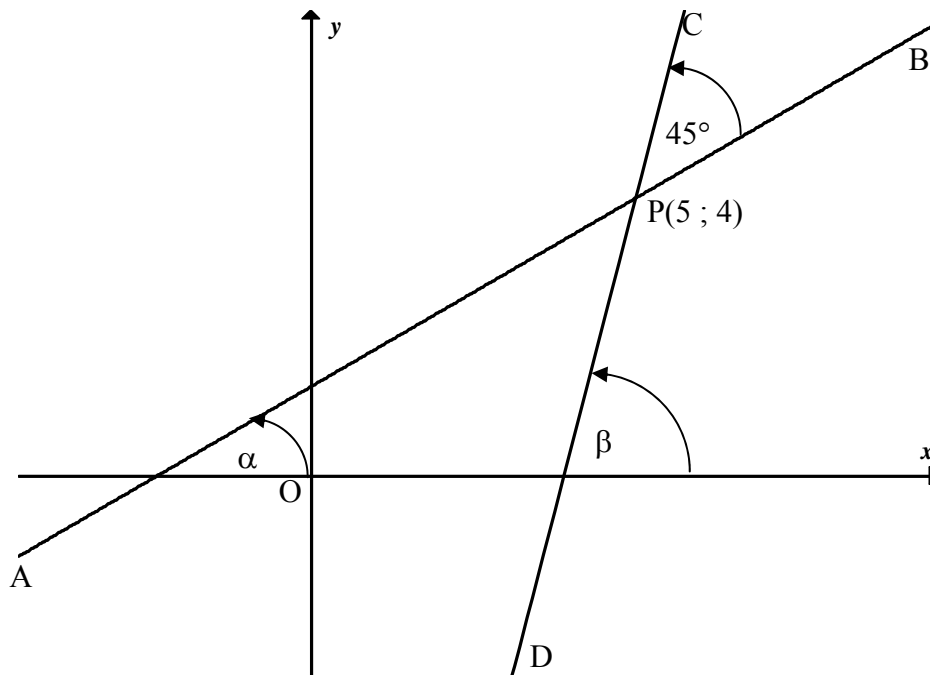
QUESTION 3

3.1	<table border="1"> <thead> <tr> <th>Time (in minutes)</th> <th>$11 \leq t < 15$</th> <th>$15 \leq t < 19$</th> <th>$19 \leq t < 23$</th> <th>$23 \leq t < 27$</th> <th>$27 \leq t < 30$</th> </tr> </thead> <tbody> <tr> <td>Frequency</td> <td>6</td> <td>9</td> <td>13</td> <td>12</td> <td>8</td> </tr> <tr> <td>Cumulative Frequency</td> <td>6</td> <td>15</td> <td>28</td> <td>40</td> <td>48</td> </tr> </tbody> </table>	Time (in minutes)	$11 \leq t < 15$	$15 \leq t < 19$	$19 \leq t < 23$	$23 \leq t < 27$	$27 \leq t < 30$	Frequency	6	9	13	12	8	Cumulative Frequency	6	15	28	40	48	✓ cumulative frequency totals (1)
Time (in minutes)	$11 \leq t < 15$	$15 \leq t < 19$	$19 \leq t < 23$	$23 \leq t < 27$	$27 \leq t < 30$															
Frequency	6	9	13	12	8															
Cumulative Frequency	6	15	28	40	48															
3.2	<p style="text-align: center;">Cumulative Frequency Curve showing the time taken to complete a task</p>	✓✓✓ plotting points at upper limits 6 correct – 3 marks 3 to 5 correct – 2 marks 1 or 2 correct – 1 mark 0 correct – 0 marks ✓ curve (4)																		
3.3	Median value at position 24. Reading off the ogive gives Median \approx 22 minutes LQ value at position 12. Lower quartile \approx 18 minutes (from ogive) UQ value at position 36. Upper quartile \approx 25,5 minutes (from ogive) NOTE: Allow margin of error for reading off the graph.	✓ median ✓ lower quartile ✓ upper quartile (3)																		
3.4		✓ box ✓ whiskers (2)																		
3.5	The times are skewed to the right. A small number of people finished this task very quickly whilst others took more time.	✓ skewed to the right (1) [11]																		

QUESTION 4

4.1	$m_{PQ} = \frac{2-0}{0-4} = -\frac{1}{2}$	✓ substitution (1)
4.2	A: $\left(\frac{0+4}{2}; \frac{2+0}{2}\right)$ A (2 ; 1)	✓ x-coordinate ✓ y-coordinate (2)
4.3	$m_{AB} \cdot m_{PQ} = -1$ $m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2$ Equation of AB is $y = 2x + c$ $\therefore 1 = 2(2) + c$ $c = -3$ Equation of AB is $y = 2x - 3$. OR $m_{AB} \cdot m_{PQ} = -1$ $m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2$ $y - 1 = 2(x - 2)$ $y - 1 = 2x - 4$ $y = 2x - 3$	✓ $m_{AB} \cdot m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ equation of AB ✓ $y = 2x - 3$ ✓ $c = -3$ (5) ✓ $m_{AB} \cdot m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ gradient of AB ✓ substitution into formula ✓ equation of AB (5)
4.4	B is the point (0 ; -3) $BQ = \sqrt{(0-4)^2 + (-3-0)^2}$ $= 5$	✓ coordinates of B ✓ substitution ✓ answer (3)
4.5	$BP = \sqrt{(0-0)^2 + (-3-2)^2}$ $= 5$ BP = BQ $\therefore \Delta BPQ$ is isosceles. OR BP = 2 + 3 $= 5$ BP = BQ $\therefore \Delta BPQ$ is isosceles	✓ BP = 5 ✓ BP = BQ (2) ✓ BP = 5 ✓ BP = BQ (2)
4.6	If PBQR is a rhombus then A is the midpoint of BR. Let the coordinates of R be (x ; y) $\frac{x+0}{2} = 2$ and $\frac{y-3}{2} = 1$ $x = 4$ $y = 5$ $\therefore R(4 ; 5)$ OR RQ PB so $x_R = 4$ RQ = PB = 5, so $y_R = 5$ $\therefore R(4 ; 5)$	✓ A is the midpoint of BR ✓ x coordinate ✓ y coordinate (3) ✓ RQ PB ✓ x coordinate ✓ y coordinate (3) [16]

QUESTION 5



<p>5.1</p> <p>AB is defined as $5y - 3x - 5 = 0$ which can be written as $y = \frac{3}{5}x + 1$</p> <p>$m_{AB} = \frac{3}{5}$</p> <p>Let α be the inclination of AB.</p> <p>$\tan \alpha = \frac{3}{5}$</p> <p>$\alpha = 30,96^\circ$.</p> <p>Let β be the inclination of CD</p> <p>$\beta = 45^\circ + 30,96^\circ$</p> <p>$= 75,96^\circ$</p> <p>Gradient of CD = $\tan 75,96^\circ = 4$.</p> <p>OR</p> <p>$\tan \beta = \tan(\alpha + 45^\circ)$</p> $= \frac{\tan \alpha + \tan 45^\circ}{1 - \tan \alpha \cdot \tan 45^\circ}$ $= \frac{\frac{3}{5} + 1}{1 - \frac{3}{5} \times 1}$ $= 4$ <p>$m_{CD} = \tan \beta$</p> <p>$m_{CD} = 4$</p>	<p>$\checkmark m_{AB} = \frac{3}{5}$</p> <p>$\checkmark \tan \alpha = \frac{3}{5}$</p> <p>$\checkmark \alpha = 30,96^\circ$</p> <p>$\checkmark \beta = 75,96^\circ$</p> <p>$\checkmark$ gradient of CD</p> <p style="text-align: right;">(5)</p> <p>\checkmark expansion</p> <p>$\checkmark \tan 45^\circ = 1$</p> <p>$\checkmark \tan \alpha = \frac{3}{5}$</p> <p>$\checkmark$ substitution</p> <p>\checkmark answer</p> <p style="text-align: right;">(5)</p>
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5.2	<p>Equation of CD is $y = 4x + c$ $\therefore 4 = 4(5) + c$ $c = -16$ Equation of CD is $y = 4x - 16$.</p> <p>OR</p> <p>$y - 4 = 4(x - 5)$ $y - 4 = 4x - 20$ $y = 4x - 16$</p>	<p>✓ y- intercept ✓ equation of CD (2)</p> <p>✓ substitution ✓ equation of CD (2)</p> <p>[7]</p>
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QUESTION 6

6.1	<p>$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x + 4)^2 + (y + 2)^2 = 58$ Centre is $(-4 ; -2)$ and the radius is $\sqrt{58}$</p>	<p>✓ completing the square (both or one) ✓ factor form ✓ centre ✓ radius (4)</p>
6.2	<p>Centre of second circle is $(4 ; 6)$ Distance between centres is $\sqrt{(4 + 4)^2 + (6 + 2)^2} = \sqrt{128} = 11,31$</p>	<p>✓ centre ✓ distance (2)</p>
6.3	<p>Sum of radii = $\sqrt{58} + \sqrt{26} = 12,71$ Distance between centres is 11,31. sum of the radii > distance between the centres \therefore the circles must overlap and hence the circles must intersect.</p>	<p>✓✓ sum of radii ✓ conclusion (3)</p>
6.4	<p>Equation of second circle: $(x - 4)^2 + (y - 6)^2 = 26$ $x^2 - 8x + 16 + y^2 - 12y + 36 = 26$ $x^2 - 8x + y^2 - 12y + 26 = 0$</p> <p>Let $(x ; y)$ be either of the two points on intersection. Then $x^2 + y^2 + 8x + 4y - 38 = 0$ and $x^2 + y^2 - 8x - 12y + 26 = 0$</p> <p>Subtract $\frac{\quad}{\quad}$ $16y + 16x - 64 = 0$ $y = -x + 4$</p> <p>Both points of intersection lie on this line. $\therefore y = -x + 4$ is the equation of the common chord.</p> <p>OR</p>	<p>✓ equation of circle in form = 0 ✓ statement – two points of intersection ✓ subtracting ✓ simplification (4)</p>

	<p>Check that the line $y = -x + 4$ cuts the two circles at the same points:</p> $(x - 4)^2 + (-x - 2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0$ $x = 3 \text{ or } x = -1$ $x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4 - x)^2 + 8x + 4(4 - x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3 \text{ or } x = -1$	<p>✓ substitution</p> <p>✓ answer</p> <p>✓ substitution</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">[13]</p>
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QUESTION 7

7.1.1	$P'(5; -2)$	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
7.1.2	$P'(5; 2)$	<p>✓ x coordinate</p> <p>✓ y coordinate</p> <p style="text-align: right;">(2)</p>
7.2.1	$K \rightarrow K'' : (14; 4) \rightarrow (2; 2)$ $U \rightarrow U'' : (18; 6) \rightarrow (3; 9)$ $H \rightarrow H'' : (16; 8) \rightarrow (4; 8)$ $L \rightarrow L'' : (18; 10) \rightarrow (5; 9)$ $E \rightarrow E'' : (14; 12) \rightarrow (6; 7)$ So “halve” and ‘interchange” or ‘interchange” and “halve”. Reflection across $y = x$ followed by contraction by $\frac{1}{2}$ OR Contraction by $\frac{1}{2}$ followed by reflection across $y = x$.	<p>✓ reflected</p> <p>✓ the line $y = x$</p> <p>✓ enlarged</p> <p>✓ scale factor of $\frac{1}{2}$</p> <p style="text-align: right;">(4)</p>
7.2.2	$H' = \frac{1}{2}(16; 8) = (8; 4)$ OR $H'(8; 16)$	<p>✓ (8 ; 4)</p> <p>✓ (8 ; 16)</p> <p style="text-align: right;">(2)</p>
7.2.3	Area KUHLE : Area $K''U''H''L''E'' = \left(\frac{2}{1}\right)^2 = 4 : 1$	<p>✓ ✓ answer</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;">[11]</p>

QUESTION 8

<p>8.1</p>	<p>For anti-clockwise rotation:</p> $x' = x \cos \theta - y \sin \theta$ $= 3 \cos 120^\circ - 2 \sin 120^\circ$ $= 3(-\cos 60^\circ) - 2 \sin 60^\circ$ $= 3\left(-\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-3 - 2\sqrt{3}}{2}$ $y' = x \sin \theta + y \cos \theta$ $= 3 \sin 120^\circ + 2 \cos 120^\circ$ $= 3 \sin 60^\circ + 2(-\cos 60^\circ)$ $= 3\left(\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2}\right)$ $= \frac{3\sqrt{3} - 2}{2}$ $P\left(\frac{-3 - 2\sqrt{3}}{2}; \frac{3\sqrt{3} - 2}{2}\right)$	<p>✓ formula</p> <p>✓ simplification ✓ substitution</p> <p>✓ answer</p> <p>✓ simplification</p> <p>✓ answer</p> <p>(6)</p>
<p>8.2</p>	$-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ $-4 = -x - \sqrt{3}y \quad \dots\dots \text{equation 1}$ $0 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(-\frac{1}{2}\right)$ $0 = \sqrt{3}x + y$ $y = -\sqrt{3}x \quad \dots\dots \text{equation 2}$ <p>Substitute equation 2 into equation 1</p> $-4 = -x - \sqrt{3}(-\sqrt{3}x)$ $-4 = -x + 3x$ $-4 = 2x$ $x = -2$ $y = 2\sqrt{3}$ $Q(-2; 2\sqrt{3})$	<p>✓ $-4 = -x - \sqrt{3}y$</p> <p>✓ $y = -\sqrt{3}x$</p> <p>✓ x-coordinate ✓ y-coordinate</p> <p>(4) [10]</p>

QUESTION 10

10.1.1	$\sin 48^\circ = \sin(36^\circ + 12^\circ)$ $= \sin 36^\circ \cos 12^\circ + \cos 36^\circ \sin 12^\circ$ $= p + q$	✓ writing 48° in terms of 36° and 12° ✓ expansion ✓ answer (3)
10.1.2	$\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$ <p>OR</p> $\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$	✓ writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24^\circ = p - q$ (3) ✓ writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24^\circ = p - q$ (3)
10.1.3	$\sin 48^\circ = 2 \sin 24^\circ \cos 24^\circ$ $\therefore p + q = 2(p - q) \cos 24^\circ$ $\therefore \cos 24^\circ = \frac{p + q}{2(p - q)}$ <p>OR</p> $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ $\therefore \cos 24^\circ = \sqrt{\frac{1 + \cos 48^\circ}{2}} = \sqrt{\frac{1}{2} \left(1 + \sqrt{1 - \sin^2 48^\circ} \right)}$ $= \sqrt{\frac{1}{2} \left(1 + \sqrt{1 - (p + q)^2} \right)}$ <p>OR</p> $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ $\cos^2 24^\circ = 1 - (p - q)^2$ $\cos 24^\circ = \sqrt{1 - (p - q)^2}$	✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ ✓ $\sin 48^\circ = p + q$ ✓ answer (3) ✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ ✓ $\sin 24^\circ = p - q$ ✓ answer (3) ✓ $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ ✓ $\sin 24^\circ = p - q$ ✓ answer (3)

<p>10.2</p> $\begin{aligned} & \sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ \\ &= \sin^2 20^\circ + (\sin(60^\circ - 20^\circ))^2 + (\sin(60^\circ + 20^\circ))^2 \\ &= \sin^2 20^\circ + (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)^2 + (\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ)^2 \\ &= \sin^2 20^\circ + \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ\right)^2 + \left(\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ\right)^2 \\ &= \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ - \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ \\ &\quad + \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ \\ &= \sin^2 20^\circ + \frac{3}{2} \cos^2 20^\circ + \frac{1}{2} \sin^2 20^\circ \\ &= \frac{3}{2} (\sin^2 20^\circ + \cos^2 20^\circ) \\ &= \frac{3}{2} \end{aligned}$ <p>OR</p> <p>Use $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$</p> <p><i>LHS</i></p> $\begin{aligned} &= \frac{3}{2} - \frac{1}{2} \{(\cos 40^\circ + \cos 80^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(\cos 60^\circ \cdot \cos 40^\circ + \sin 60^\circ \sin 40^\circ + \cos 60^\circ \cdot \cos 40^\circ - \sin 60^\circ \sin 40^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(2 \cos 60^\circ \cos 20^\circ) - \cos 20^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \left\{ \left(2 \times \frac{1}{2} \cos 20^\circ \right) - \cos 20^\circ \right\} \\ &= \frac{3}{2} - 0 \\ &= \frac{3}{2} \end{aligned}$	<p>✓40°= 60° - 20° ✓80°= 60° + 20° ✓ ✓expansions ✓ substitution</p> <p>✓ simplification ✓ factorisation</p> <p>(7)</p> <p>✓40°= 60° - 20° ✓80°= 60° + 20° ✓ expansion of cos 40° ✓ expansion of cos 60° ✓ simplification ✓ simplification ✓ answer for bracket</p> <p>(7)</p>
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<p>10.3.1</p>	$\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x}$ $= \frac{\sin^2 x(\sin^2 x + \cos^2 x)}{1 + \cos x}$ $= \frac{\sin^2 x}{1 + \cos x}$ $= \frac{1 - \cos^2 x}{1 + \cos x}$ $= \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)}$ $= 1 - \cos x$	<p>✓ factorisation</p> <p>✓ $\sin^2 x + \cos^2 x = 1$</p> <p>✓ identity</p> <p>✓ factorisation</p> <p>(4)</p>
<p>10.3.2</p>	<p>$1 + \cos x = 0$</p> <p>$\cos x = -1$</p> <p>$x = 180^\circ + k.360^\circ; k \in Z$</p> <p>Undefined for $x = 180^\circ + k.360^\circ; k \in Z$.</p>	<p>✓ $1 + \cos x = 0$</p> <p>✓ $180^\circ + k.360^\circ$</p> <p>(2)</p> <p>[22]</p>

QUESTION 11

<p>11.1</p>	<p>$1 + \sin x = \cos 2x$</p> <p>$1 + \sin x = 1 - 2\sin^2 x$</p> <p>$\sin x + 2\sin^2 x = 0$</p> <p>$\sin x(1 + 2\sin x) = 0$</p> <p>$\sin x = 0$ or $\sin x = -\frac{1}{2}$,</p> <p>$x = k.180$ or $x = -30^\circ + k.360$ $k \in Z$</p> <p>$x = 210^\circ + k.360$</p> <p>$x \in \{180^\circ; 210; 330^\circ; 360^\circ\}$</p> <p>OR</p> <p>$1 + \sin x = \cos 2x$</p> <p>$1 + \sin x = \cos^2 x - \sin^2 x$</p> <p>$1 + \sin x = 1 - \sin^2 x - \sin^2 x$</p> <p>$\sin x + 2\sin^2 x = 0$</p> <p>$\sin x(1 + 2\sin x) = 0$</p> <p>$\sin x = 0$ or $\sin x = -\frac{1}{2}$,</p> <p>$x = k.180$ or $x = -30^\circ + k.360$ $k \in Z$</p> <p>$x = 210^\circ + k.360$</p> <p>$x \in \{180^\circ; 210; 330^\circ; 360^\circ\}$</p>	<p>✓ expansion</p> <p>✓ factorisation</p> <p>✓ equations</p> <p>✓ $x = k.180$</p> <p>✓ solution for $\sin x = -\frac{1}{2}$,</p> <p>✓✓ answers</p> <p>(7)</p> <p>✓ expansion</p> <p>✓ factorisation</p> <p>✓ equations</p> <p>✓ $x = k.180$</p> <p>✓ solution for $\sin x = -\frac{1}{2}$,</p> <p>✓✓ answers</p> <p>(7)</p>
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<p>11.2</p>		<p> $1 + \sin x$ ✓ max and min values ✓ shape $\cos 2x$ ✓ amplitude ✓ intercepts </p> <p style="text-align: right;">(4)</p>
<p>11.3</p>	<p>$180^\circ \leq x \leq 210^\circ$ or $330^\circ \leq x \leq 360^\circ$</p>	<p>✓✓✓ answer</p> <p style="text-align: right;">(3) [14]</p>

QUESTION 12

<p>12.1</p>	$\frac{b}{\sin[180^\circ - (\alpha + \beta)]} = \frac{BC}{\sin \alpha}$ $BC \sin(\alpha + \beta) = b \sin \alpha$ $BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ <p>but $BC = DF$</p> $\therefore DF = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ $\cos \theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos \theta}$ $\therefore DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$	<p> ✓ sine rule ✓ $\hat{A}BC = 180^\circ - (\alpha + \beta)$ ✓ $BC = \dots$ ✓ $BC = DF$ ✓ manipulation ✓ $DE = \dots$ </p> <p style="text-align: right;">(6)</p>
<p>12.2</p>	$DE = \frac{2000 \sin 43^\circ}{\sin 79^\circ \cdot \cos 27^\circ}$ $= 1559,50 \text{ m}$	<p> ✓ substitution numerator ✓ substitution denominator ✓ answer </p> <p style="text-align: right;">(3) [9]</p>

TOTAL: 150



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2010

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 4 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Round off to TWO decimal places if necessary, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. FOUR diagram sheets for QUESTION 1.2, QUESTION 2.1, QUESTION 2.2, QUESTION 7.1 and QUESTION 12.1 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of this question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and present your work neatly.

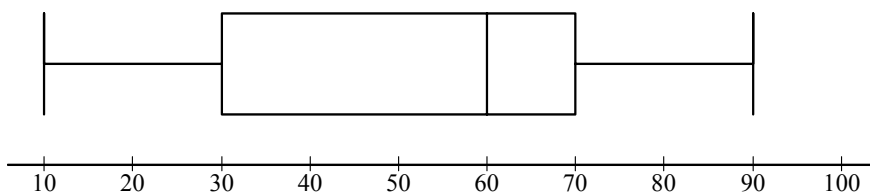
QUESTION 1

Two Mathematics classes, A and B, are in competition to see which class performed best in the June examination. The marks of the learners in Class A are given below and the box and whisker diagram below illustrates the results of Class B. Both classes have 25 learners. (Marks are given in %.)

The marks of the learners in Class A are:

9	14	14	19	21
23	33	35	37	37
42	45	55	56	57
59	68	75	75	75
77	78	80	81	92

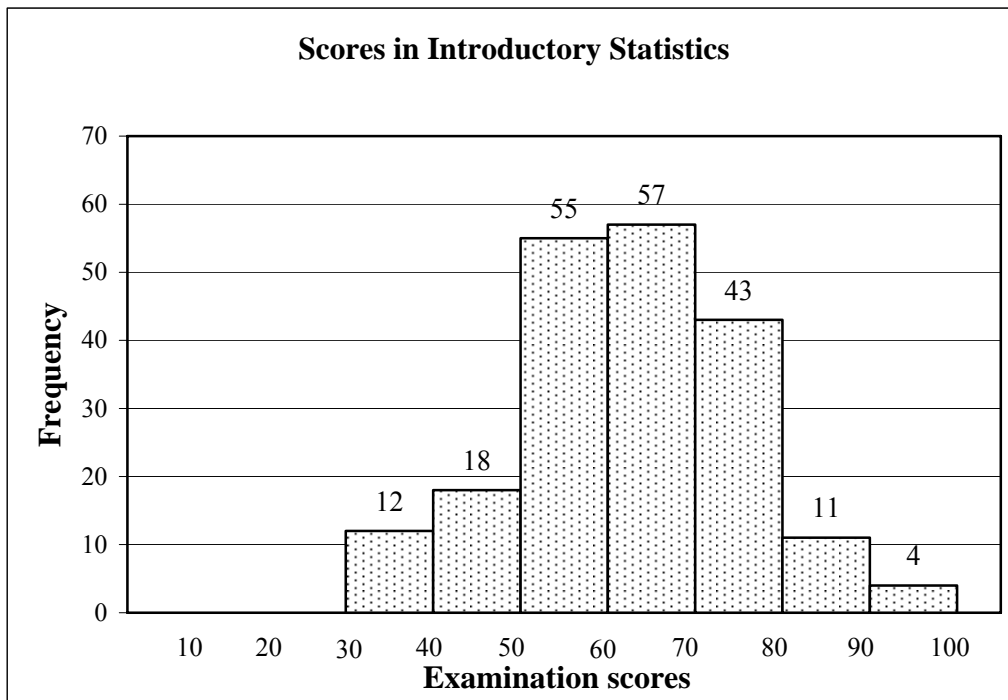
The box and whisker diagram for the learners in Class B is:



- 1.1 Write down the five-number summary for Class A. (4)
 - 1.2 Draw the box and whisker diagram that represents Class A's marks on DIAGRAM SHEET 1. Clearly indicate ALL relevant values. (2)
 - 1.3 Determine which class performed better in the June examination and give reasons for your conclusion. (3)
- [9]**

QUESTION 2

The histogram below shows the distribution of examination scores for 200 learners in Introductory Statistics.



- 2.1 Complete the cumulative frequency table for the above data provided on DIAGRAM SHEET 2. (2)
 - 2.2 Draw an ogive of the above data on the grid provided on DIAGRAM SHEET 2. (5)
 - 2.3 Use the ogive to estimate how many learners scored 75% or more for the examination. (1)
- [8]**

QUESTION 3

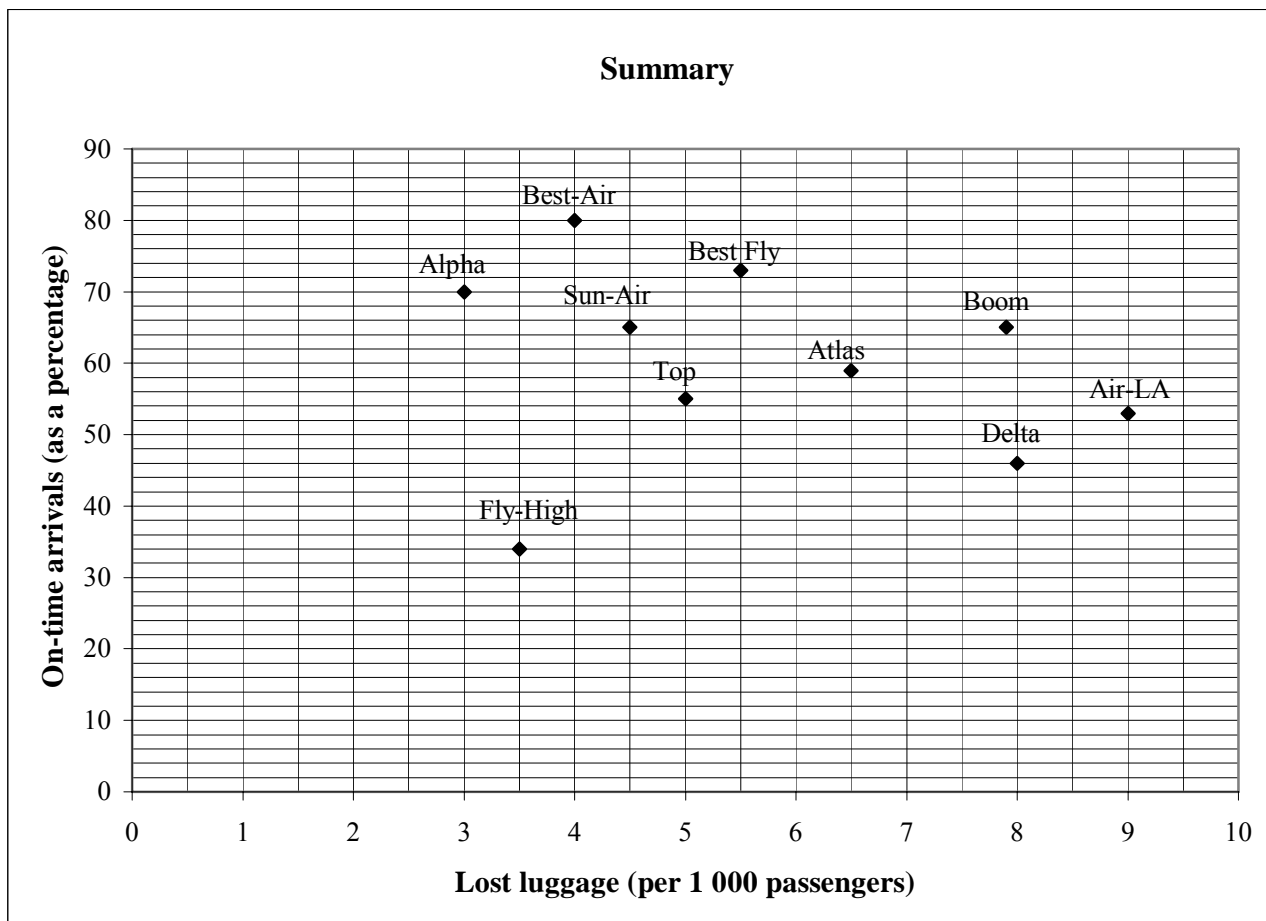
The owner of an ice-cream parlour gathered information on the average sales per day of litres of ice-cream during a festival. The table below shows a summary for 12 days.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Averages sales of ice-cream (litres)	217	211	221	239	144	161	168	185	265	249	160	184

- 3.1 Calculate the mean number of litres of ice-cream that the parlour sells per day during the festival. (2)
 - 3.2 Calculate the standard deviation of the given information. (3)
 - 3.3 What is the maximum number of litres of ice-cream that the owner must stock per day in order to be within ONE standard deviation of the mean? (2)
- [7]**

QUESTION 4

A researcher suspects that airlines, whose planes arrive on time, are less likely to lose the luggage of their passengers. Information gathered from 10 airline companies is summarised in the grid below.

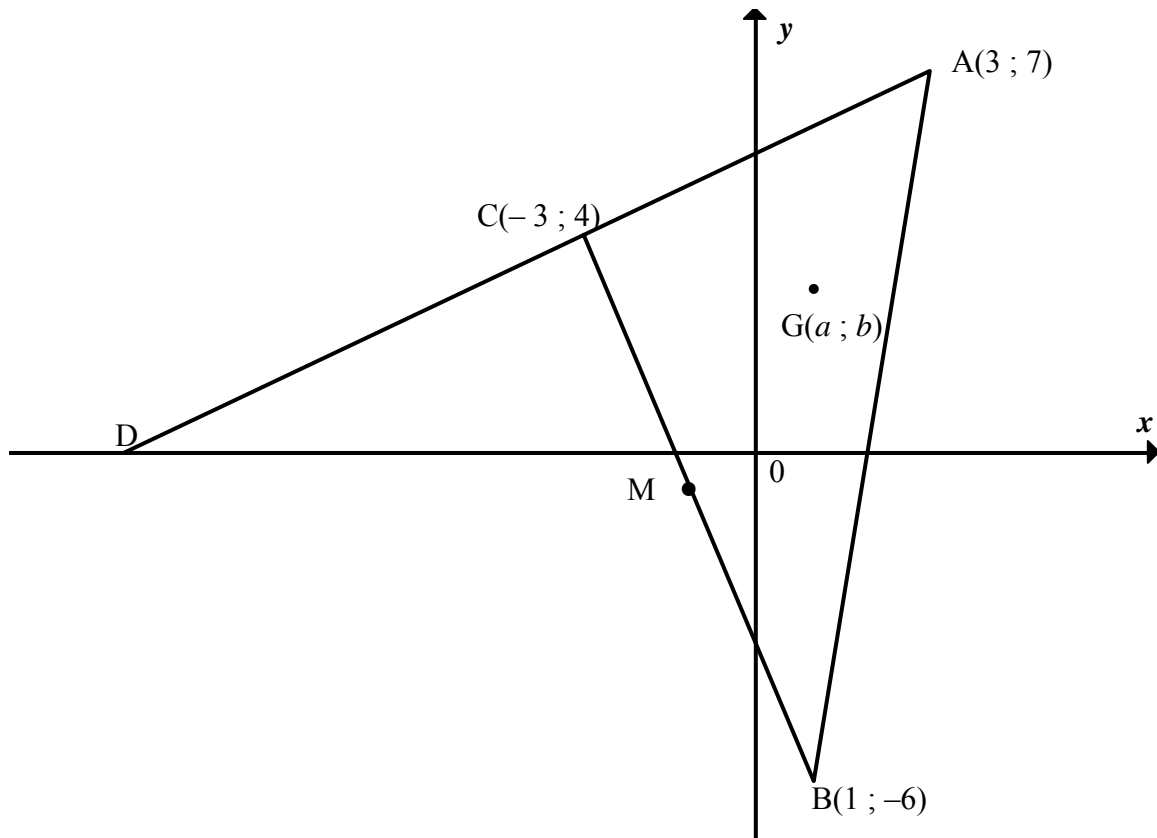


Use the scatter plot to answer the following questions.

- 4.1 Which airline has the worst record for on-time arrivals? (1)
 - 4.2 Is the following statement likely to be TRUE? Motivate your answer.
Of 5 120 passengers transported by Boom airlines, 40 passengers lost their luggage. (1)
 - 4.3 Does the data confirm the researcher’s suspicions? Justify your answer. (2)
 - 4.4 Which ONE of the 10 airlines would you prefer to use? Give a reason for your answer. (2)
- [6]**

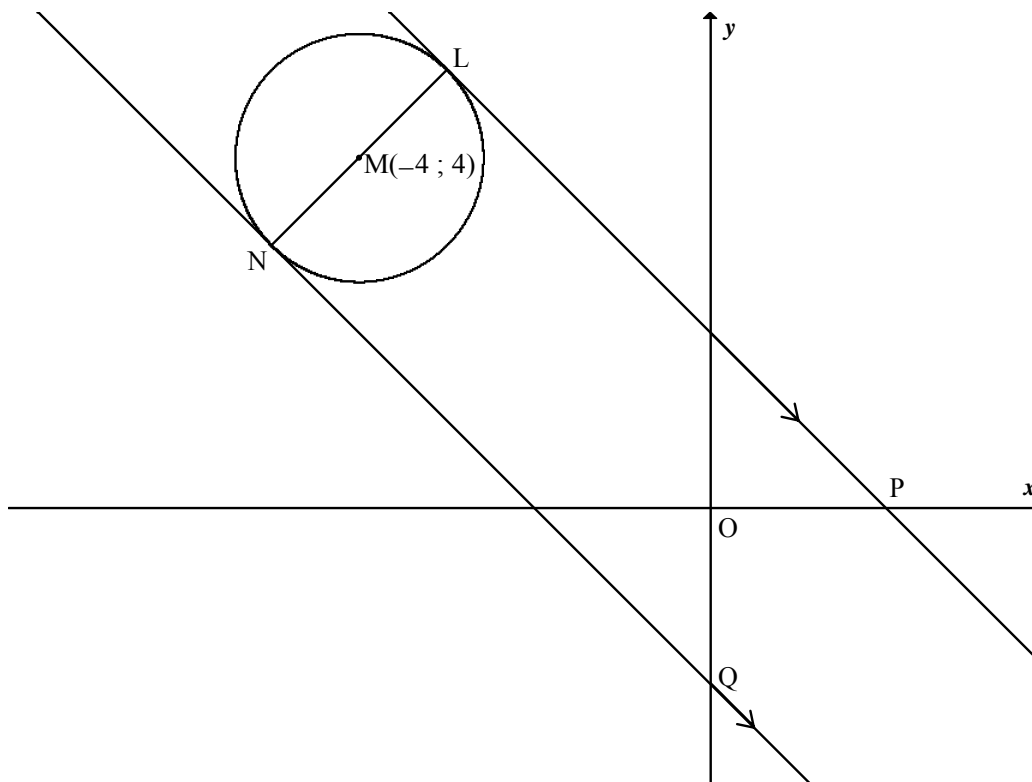
QUESTION 5

In the diagram below, A, B and C are the vertices of a triangle. AC is extended to cut the x -axis at D.



- 5.1 Calculate the gradient of:
- 5.1.1 AD (2)
- 5.1.2 BC (1)
- 5.2 Calculate the size of \hat{DCB} . (3)
- 5.3 Write down an equation of the straight line AD. (2)
- 5.4 Determine the coordinates of M, the midpoint of BC. (2)
- 5.5 If $G(a; b)$ is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$. (4)
- 5.6 Hence calculate TWO possible values of b if $GC = \sqrt{17}$. (6)
- [20]**

QUESTION 6



The line LP, with equation $y + x - 2 = 0$, is a tangent at L to the circle with centre $M(-4 ; 4)$. LN is a diameter of the circle. Also $LP \parallel NQ$, where P lies on the x -axis, and Q lies on the y -axis.

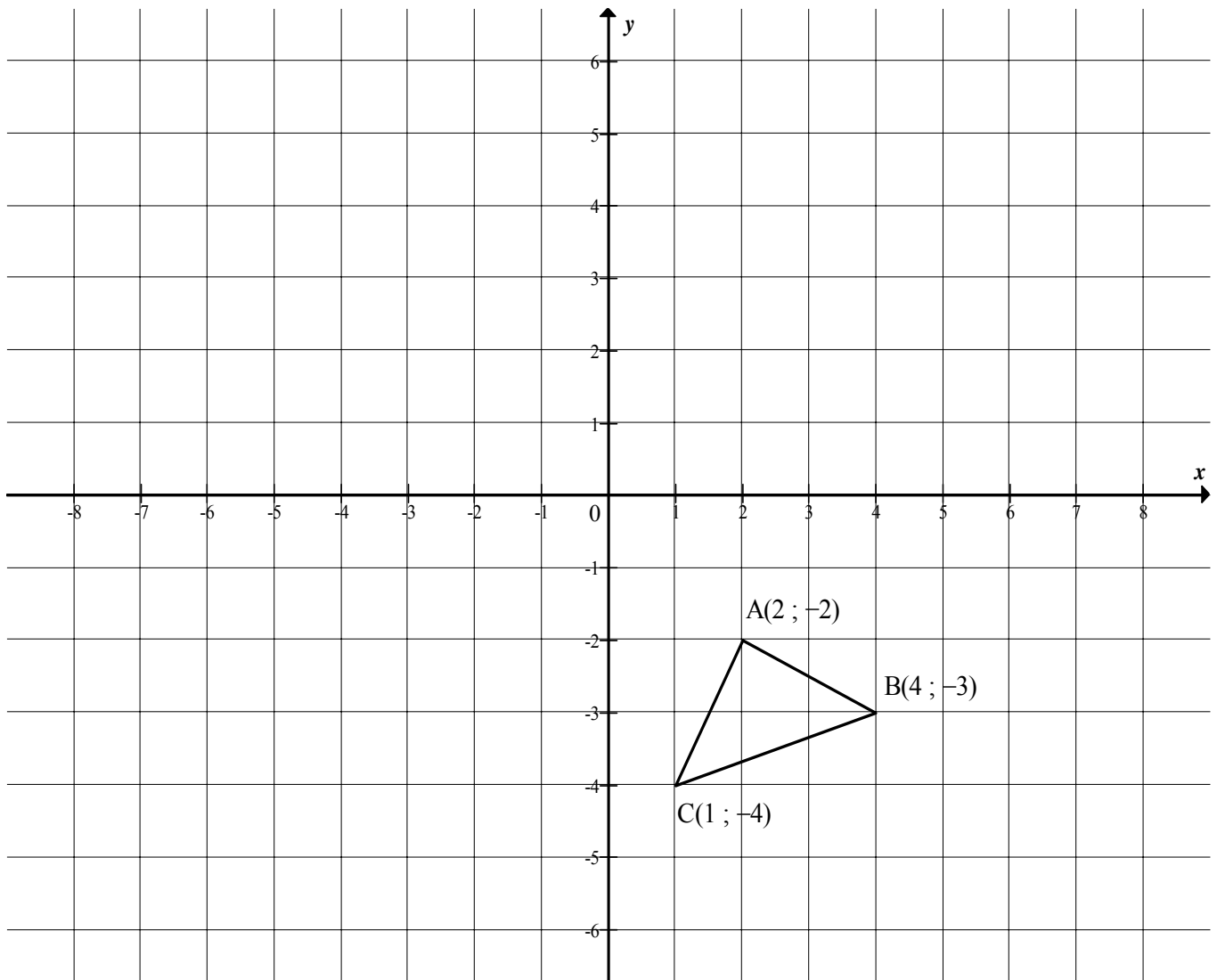
- 6.1 Determine the equation of the diameter LN. (3)
 - 6.2 Calculate the coordinates of L. (2)
 - 6.3 Determine the equation of the circle. (3)
 - 6.4 Write down the coordinates of N. (3)
 - 6.5 Write down the equation of NQ. (3)
 - 6.6 If the length of the diameter is doubled and the circle is translated horizontally 6 units to the right, write down the equation of the new circle. (3)
- [17]**

QUESTION 7

A transformation T is described as follows:

- A reflection in the x -axis, followed by
- A translation of 4 units left and 2 units down, followed by
- An enlargement through the origin by a factor of 2

In the diagram $\triangle ABC$ is given with vertices $A(2 ; -2)$, $B(4 ; -3)$ and $C(1 ; -4)$.



- 7.1 If $\triangle ABC$ is transformed by T to $\triangle A'B'C'$ (in that order), use the grid provided on DIAGRAM SHEET 3 to sketch $\triangle A'B'C'$. Show ALL the steps. (6)
- 7.2 Write down the general rule for $(x ; y)$ under transformation T in the form $(x ; y) \rightarrow \dots$ (4)
- 7.3 Calculate the area of $\triangle A'B'C'$. (4)

[14]

QUESTION 8

- 8.1 The point $K(2 ; 4)$ is rotated about the origin through an angle of 75° , in an anticlockwise direction. Without the use of a calculator, determine the x -coordinate of the image K' of K . Simplify your answer. (6)
- 8.2 The point $(3 ; 1)$ is rotated in an anticlockwise direction about the origin through an angle β . If the image is $\left(\frac{3-\sqrt{3}}{2}; \frac{1+3\sqrt{3}}{2}\right)$, calculate β . (6)
- [12]**

QUESTION 9

Given: $\tan \alpha = \frac{3}{4}$; where $\alpha \in [0^\circ ; 90^\circ]$

With the use of a sketch and without the use of a calculator, calculate:

- 9.1 $\sin \alpha$ (3)
- 9.2 $\cos^2(90^\circ - \alpha) - 1$ (2)
- 9.3 $1 - \sin 2\alpha$ (3)
- [8]**

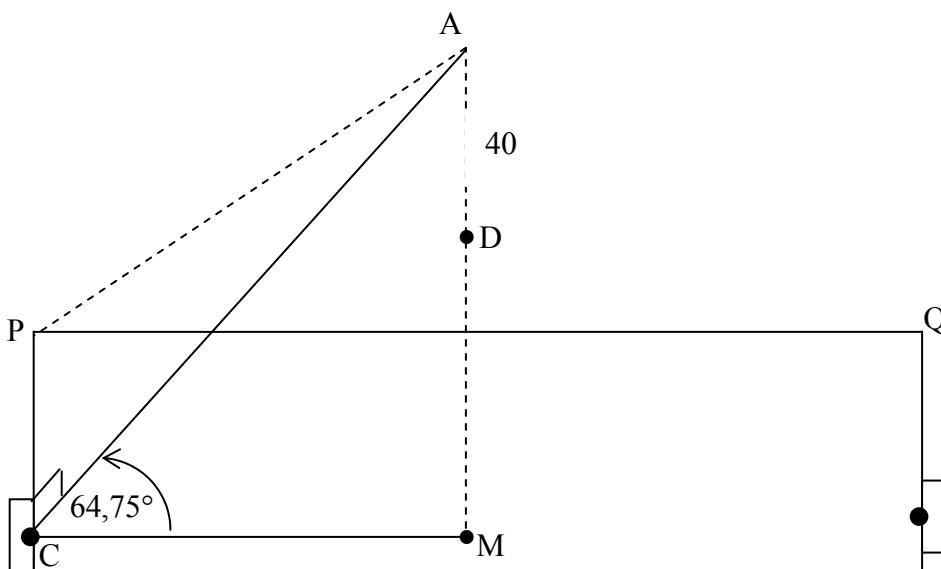
QUESTION 10

(You may NOT use a calculator to answer this question.)

- 10.1 Simplify completely:
- $$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta)\sin(-\theta)}{\sin 180^\circ - \tan 135^\circ} \quad (5)$$
- 10.2 Prove that for any angle A :
- $$\frac{4 \sin A \cos A \cos 2A \sin 15^\circ}{\sin 2A(\tan 225^\circ - 2 \sin^2 A)} = \frac{\sqrt{6} - \sqrt{2}}{2} \quad (6)$$
- 10.3 Determine the general solution of:
- $$6 \cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0 \quad (6)$$
- [17]**

QUESTION 11

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is $64,75^\circ$. The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also $AC \perp PC$. In the figure below $PQ = 100$ metres and $PC = 32$ metres.



- 11.1 Determine AC. (3)
 - 11.2 Calculate \hat{PAC} . (3)
 - 11.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C. (4)
- [10]**

QUESTION 12

Given: $f(x) = 2 \cos x$ and $g(x) = \tan 2x$

- 12.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90^\circ ; 90^\circ]$ (6)
 - 12.2 Solve for x if $2 \cos x = \tan 2x$ and $x \in [-90^\circ ; 90^\circ]$. Show ALL working details. (8)
 - 12.3 Use the graph to solve for x : $2 \cos x \cdot \tan 2x > 0$. (4)
 - 12.4 Write down the period of $f\left(\frac{x}{2}\right)$. (2)
 - 12.5 Write down the equations of the asymptotes of $g(x - 25^\circ)$, where $x \in [-90^\circ ; 90^\circ]$. (2)
- [22]**

TOTAL: 150

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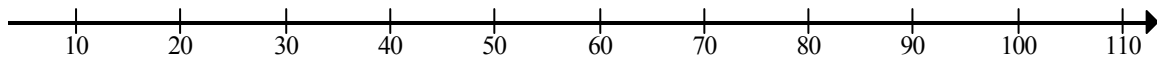
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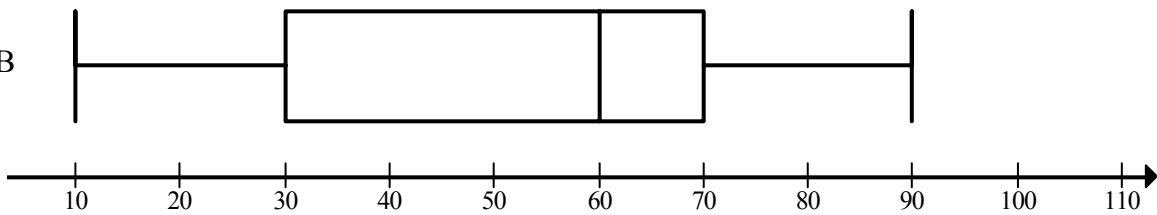
DIAGRAM SHEET 1

QUESTION 1.2

Class A



Class B



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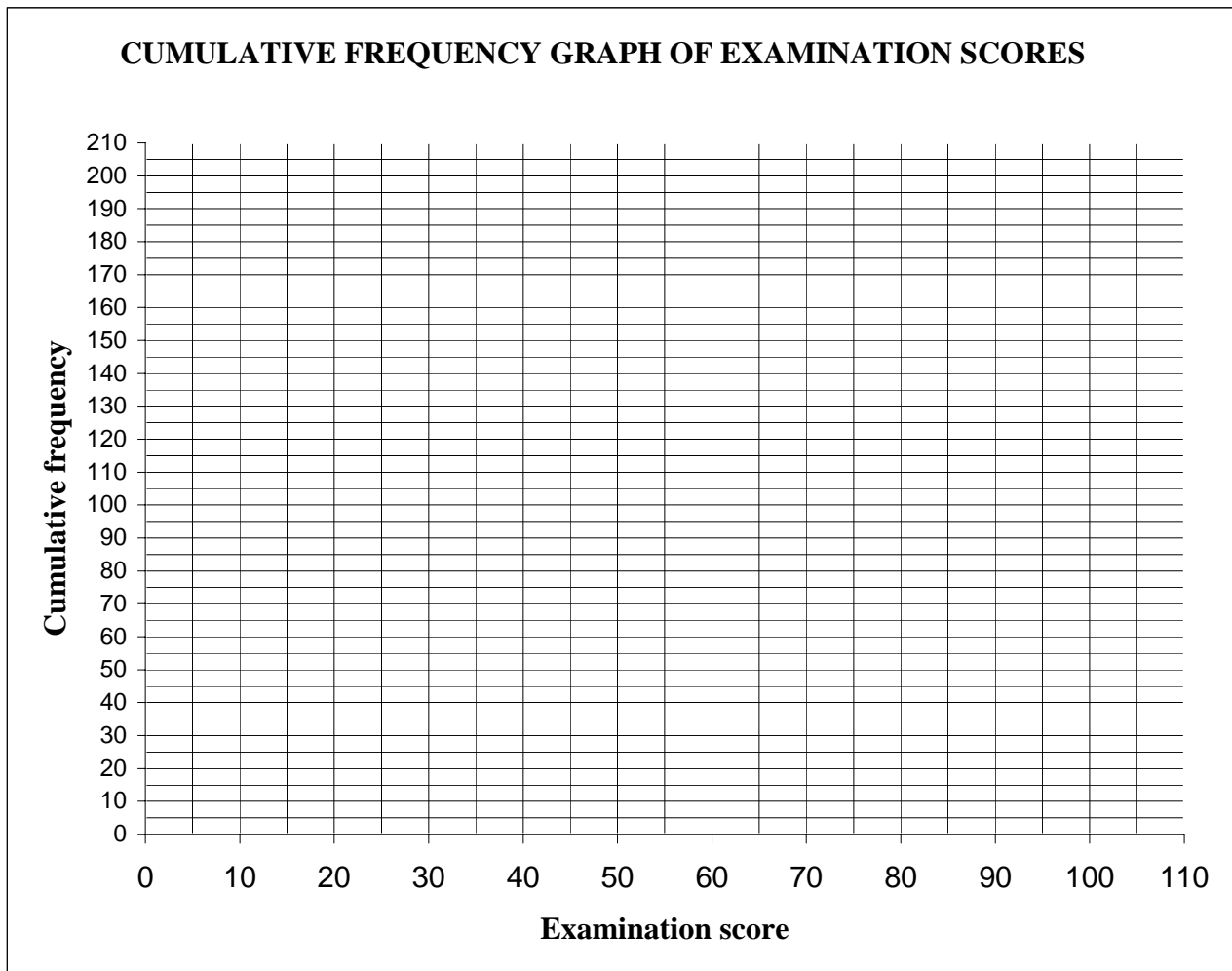
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DIAGRAM SHEET 2

QUESTION 2.1

EXAMINATION SCORE (x)	FREQUENCY	CUMULATIVE FREQUENCY
$30 \leq x < 40$	12	
$40 \leq x < 50$	18	
$50 \leq x < 60$	55	
$60 \leq x < 70$	57	
$70 \leq x < 80$	43	
$80 \leq x < 90$	11	
$90 \leq x < 100$	4	

QUESTION 2.2



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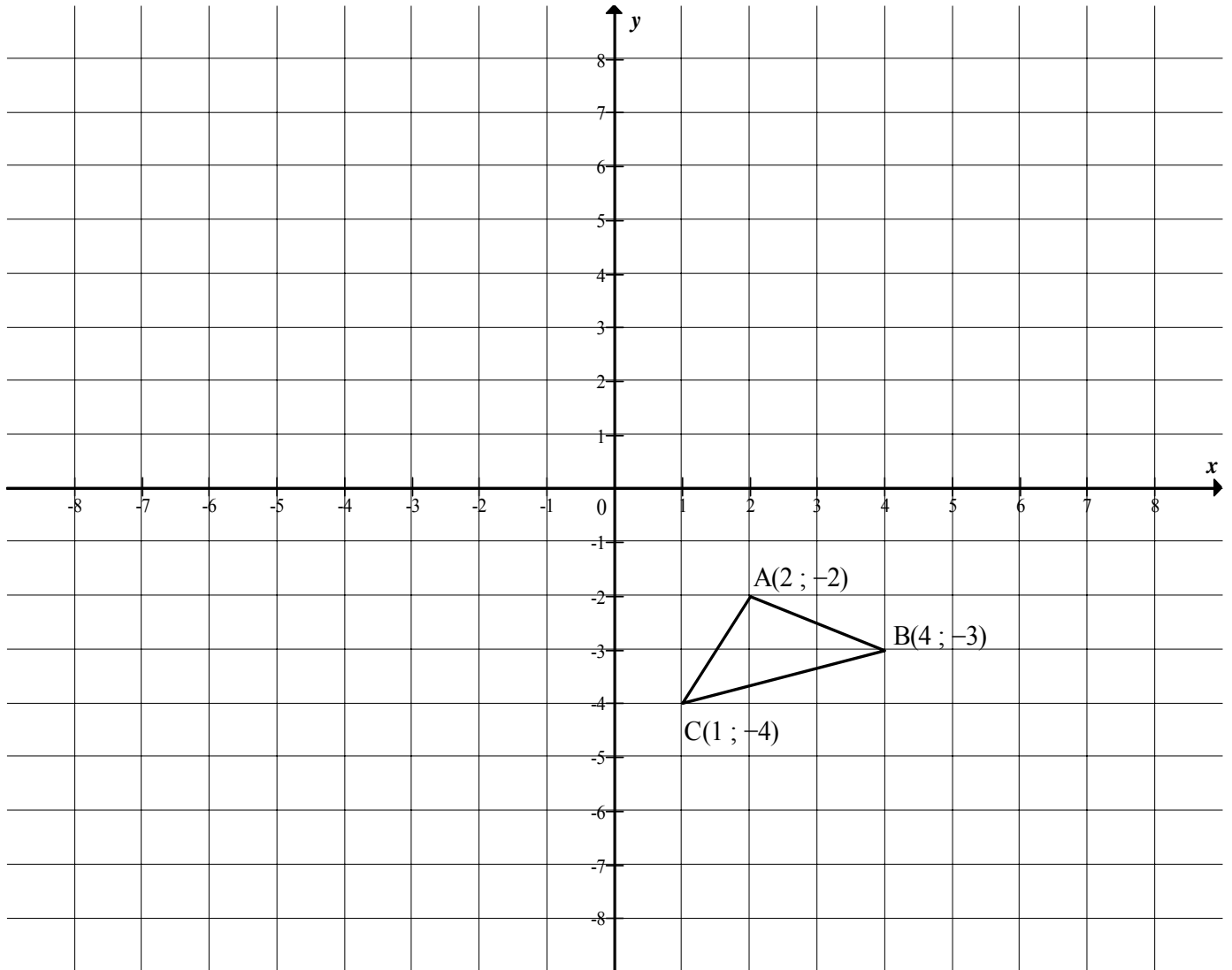
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DIAGRAM SHEET 3

QUESTION 7.1



CENTRE NUMBER:

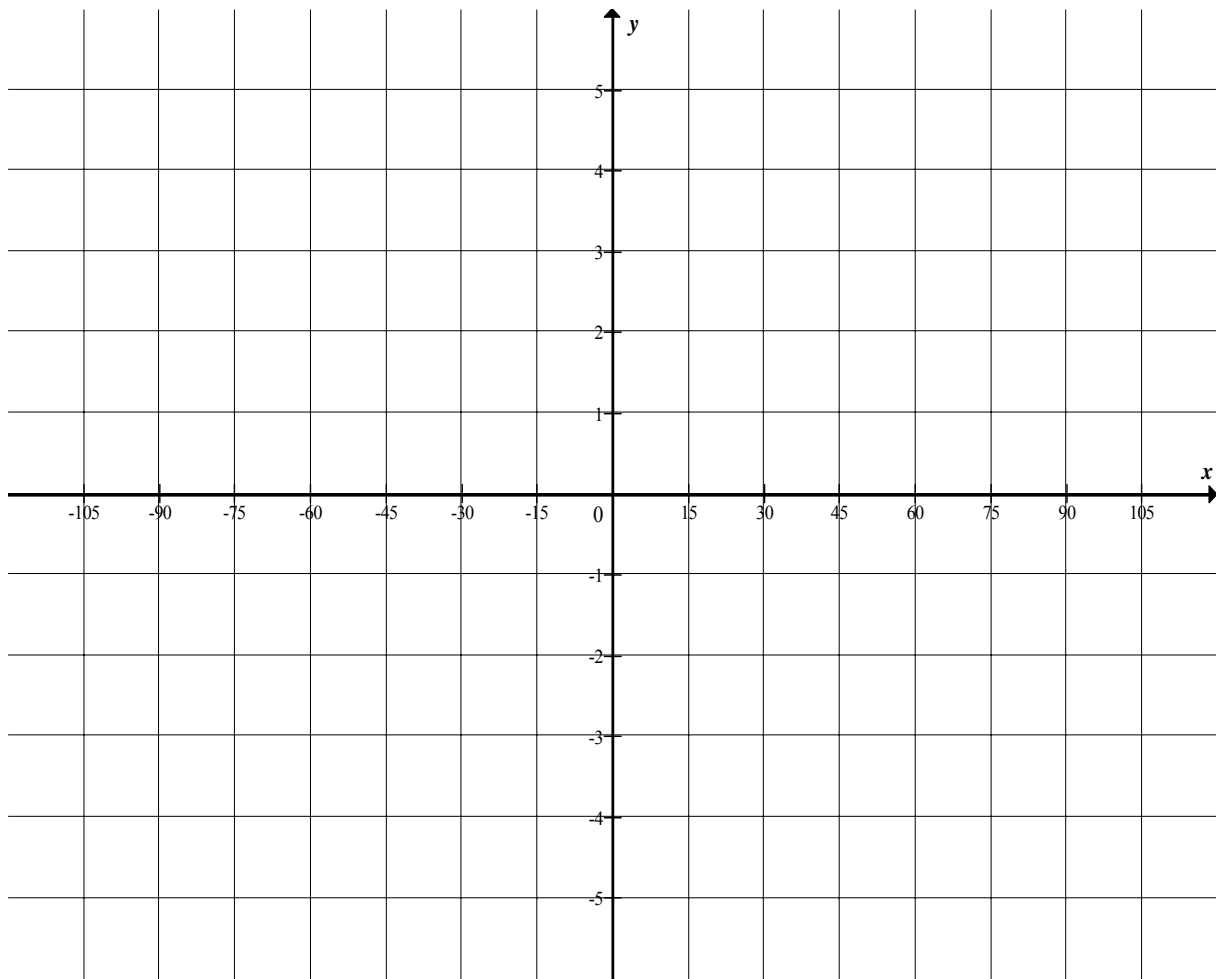
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DIAGRAM SHEET 4

QUESTION 12.1



INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2010

MARKS: 100

TIME: 2 hours

This question paper consists of 7 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round your answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. THREE diagram sheets for answering QUESTION 4.1, QUESTION 7, QUESTION 8.1, QUESTION 8.2, QUESTION 9 and QUESTION 10 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and present your work neatly.

QUESTION 1

A school organised a camp for their 103 Grade 12 learners. The learners were asked to indicate their food preferences for the camp. They had to choose from chicken, vegetables and fish.

The following information was collected:

- 2 learners do not eat chicken, fish or vegetables
- 5 learners eat only vegetables
- 2 learners only eat chicken
- 21 learners do not eat fish
- 3 learners eat only fish
- 66 learners eat chicken and fish
- 75 learners eat vegetables and fish

Let the number of learners who eat chicken, vegetables and fish be x .

- 1.1 Draw an appropriate Venn diagram to represent the information. (7)
- 1.2 Calculate x . (2)
- 1.3 Calculate the probability that a learner, chosen at random:
- 1.3.1 Eats only chicken and fish, and no vegetables. (2)
- 1.3.2 Eats any TWO of the given food choices: chicken, vegetables and fish. (2)
- [13]

QUESTION 2

A supermarket conducted a survey on its service to customers. This was done on a Wednesday morning. The survey indicated that 78% of the customers were satisfied with the service offered by the supermarket and 90% of the customers agreed that the supermarket was a stress-free place to do their shopping. The total number of customers interviewed was 130.

- 2.1 Would you agree that the supermarket could regard the findings of the survey as reliable? Motivate your answer. (2)
- 2.2 How many customers thought that the supermarket's service was not satisfactory? (2)
- 2.3 Give TWO recommendations to the supermarket on using surveys to gather information regarding its customer service. (2)
- [6]

QUESTION 3

A toothpaste manufacturer fills toothpaste tubes with an average of 182 grams of toothpaste. The standard deviation of a control sample is 0,454 grams.

- 3.1 If 20 000 tubes of toothpaste are manufactured daily, how many tubes will fall within ONE standard deviation of the mean? (2)
- 3.2 Calculate the range of the weight of toothpaste tubes in the control sample. (4)
- [6]**

QUESTION 4

The data below shows the pulse rate of a sample of 12 people when they rest and then again after 2 minutes of jogging.

Resting heart rate (beats per minute)	47	55	95	65	75	78	80	72	82	76	68	62
Heart rate after jogging (beats per minute)	65	68	100	78	81	90	85	84	105	88	75	80

- 4.1 Draw a scatter plot of the data given on the grid provided on DIAGRAM SHEET 1. (3)
- 4.2 Calculate the equation of the least squares line for this data. (4)
- 4.3 Calculate the correlation coefficient. (2)
- 4.4 Comment on the correlation of the data. (2)
- 4.5 If Joan's heart rate after jogging is 86 beats per minute, what is her resting heart rate, in beats per minute? (2)
- [13]**

QUESTION 5

In Gauteng number plates are designed with 3 alphabetical letters, excluding the 5 vowels, next to one another and then any 3 digits, from 0 to 9, next to one another. The GP is constant in all Gauteng number plates, for example TTT 012 GP. Letters and digits may be repeated in a number plate.

- 5.1 How many unique number plates are available? (3)
- 5.2 What is the probability that a car's number plate will start with a Y? (3)
- 5.3 What is the probability that a car's number plate will contain only one 7? (3)
- 5.4 How many unique number plates will be available if the letters and numbers are not repeated? (3)
- [12]**

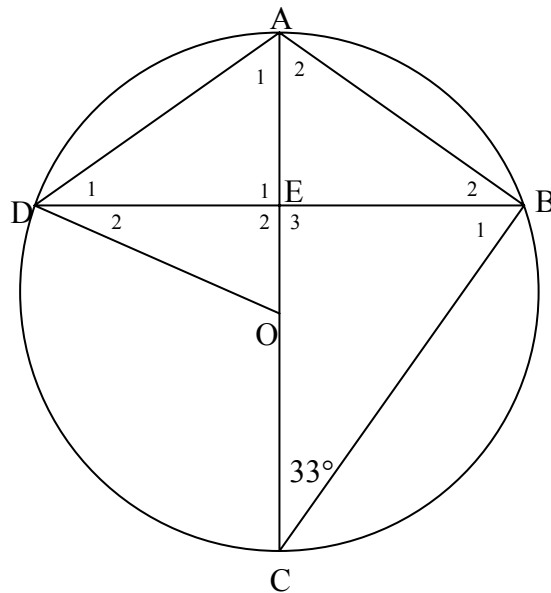
QUESTION 6

Given: $T_{k+1} = T_k + (5 - 4k)$ where $T_1 = 3$ and $k \geq 1$

- 6.1 Determine the FIRST FOUR terms of the sequence. (3)
 - 6.2 What type of sequence will this formula generate? Give a reason for your answer. (2)
- [5]**

QUESTION 7

In the diagram below AC is a diameter of the circle with centre O. AC and chord BD intersect at E. AB, BC and AD are also chords of the circle. OD is joined. $AE \perp BD$.

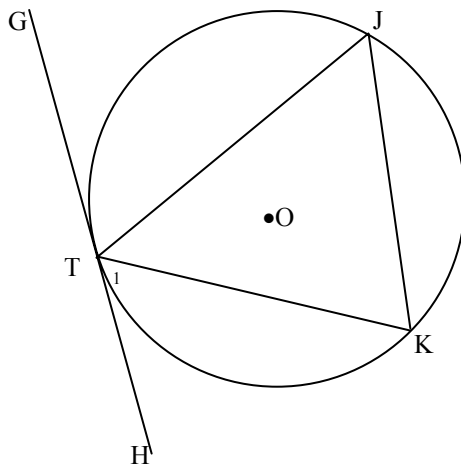


If $\hat{C} = 33^\circ$, calculate, with reasons, the size of:

- 7.1 \hat{A}_1 (3)
 - 7.2 \hat{D}_2 (2)
 - 7.3 Show that AE bisects \hat{DAB} (3)
- [8]**

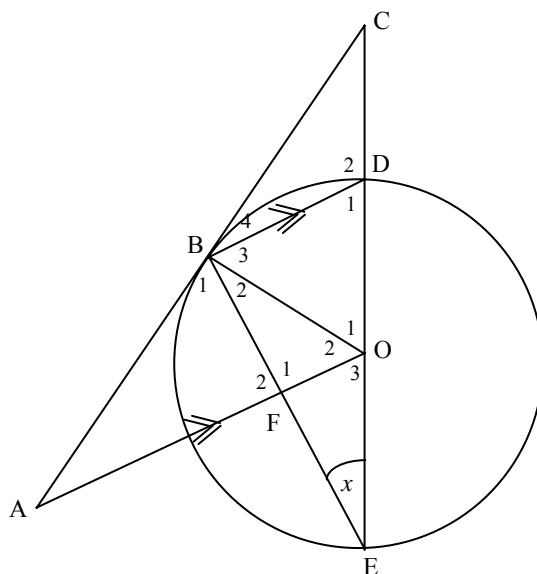
QUESTION 8

- 8.1 In the diagram below O is the centre of the circle. GH is a tangent to the circle at T. J and K are points on the circumference of the circle. TJ, TK and JK are joined.



Prove the theorem that states $\hat{T}_1 = \hat{TJK}$. (5)

- 8.2 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F. $BD \parallel AO$. $\hat{E} = x$.



- 8.2.1 Write down, with reasons, THREE other angles equal to x . (4)
- 8.2.2 Determine, with reasons, \hat{CBE} in terms of x . (3)
- 8.2.3 Prove that F is the midpoint of BE. (4)
- 8.2.4 Prove that $\triangle CBD \parallel \triangle CEB$. (2)
- 8.2.5 Prove that $2EF \cdot CB = CE \cdot BD$. (3)

[21]

QUESTION 9

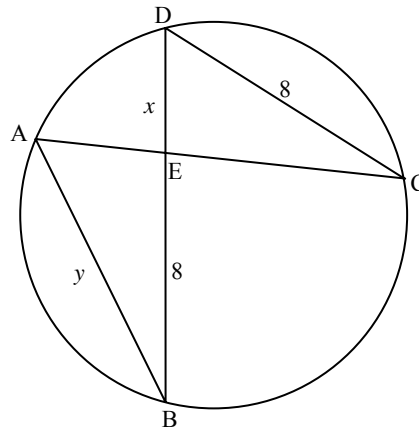
In the diagram below A, B, C and D are points on the circumference of the circle.

BD and AC intersect at E. Also,

EB = 8 cm,

DC = 8 cm and

AE : EC = 4 : 7.

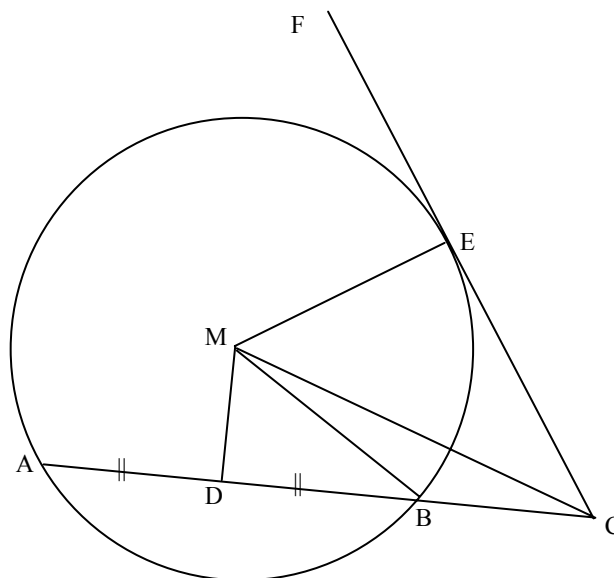


If $DE = x$ units and $AB = y$ units, calculate x and y .

[6]

QUESTION 10

In the diagram below M is the centre of the circle. FEC is a tangent to the circle at E. D is the midpoint of AB.



10.1 Prove MDCE is a cyclic quadrilateral. (3)

10.2 Prove that $MC^2 = MB^2 + DC^2 - DB^2$. (3)

10.3 Calculate CE if $AB = 60$ mm, $ME = 40$ mm and $BC = 20$ mm. (4)

[10]

TOTAL: 100

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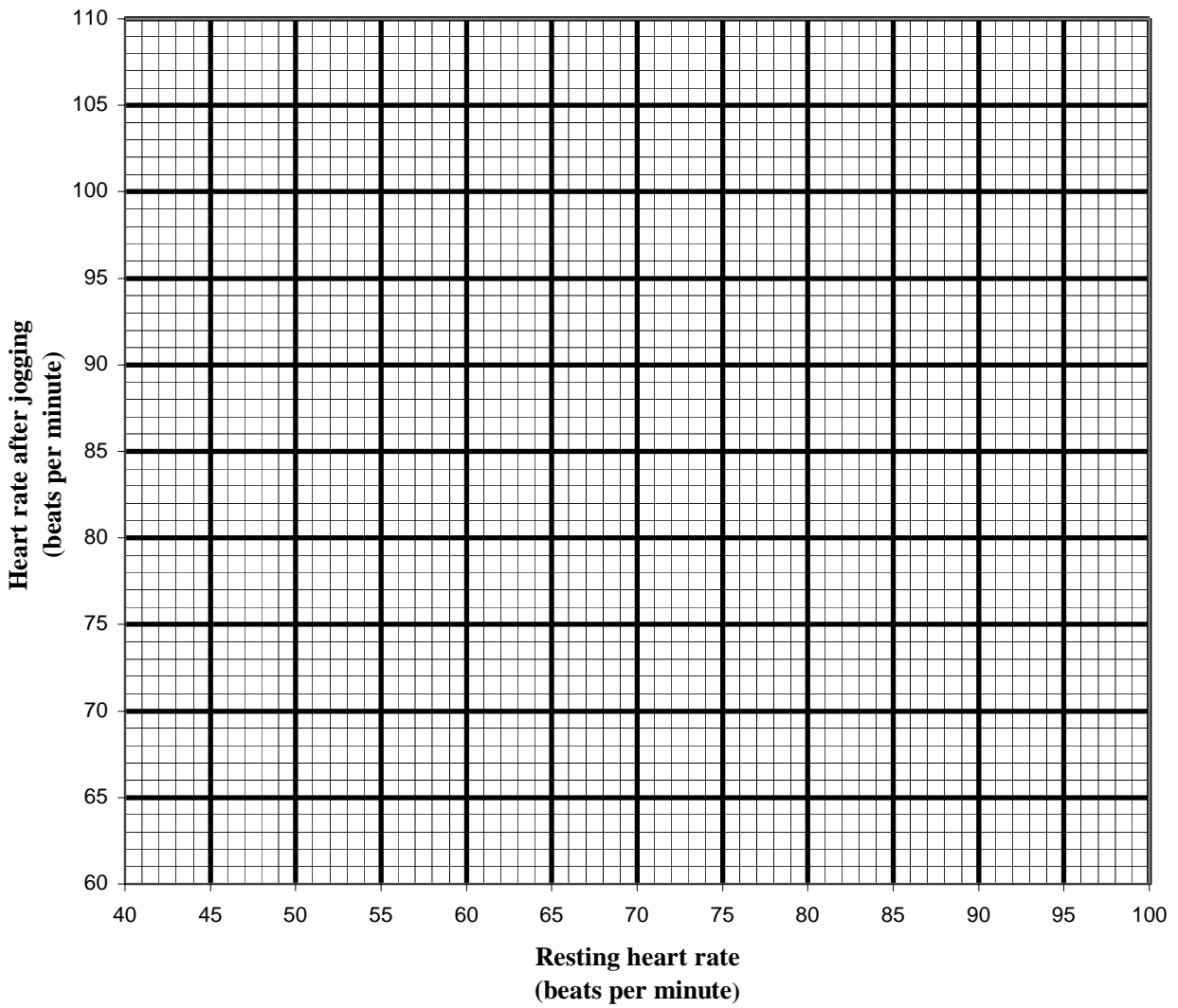
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DIAGRAM SHEET 1

QUESTION 4.1

Scatter plot showing resting heart rate vs heart rate after jogging



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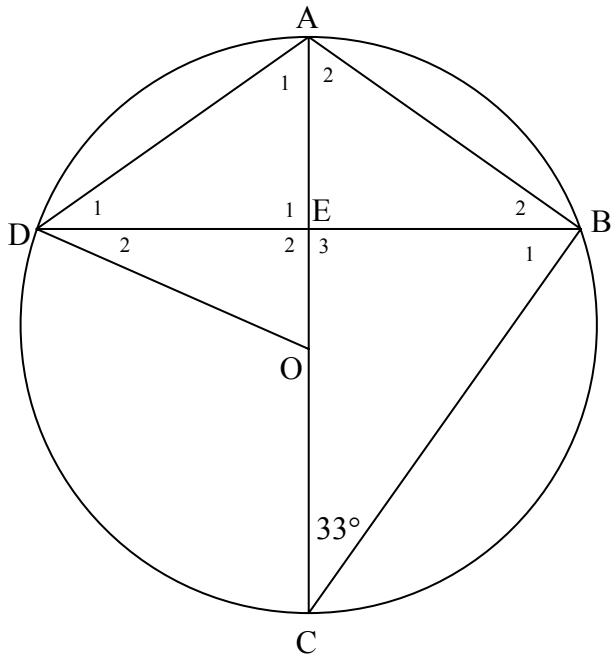
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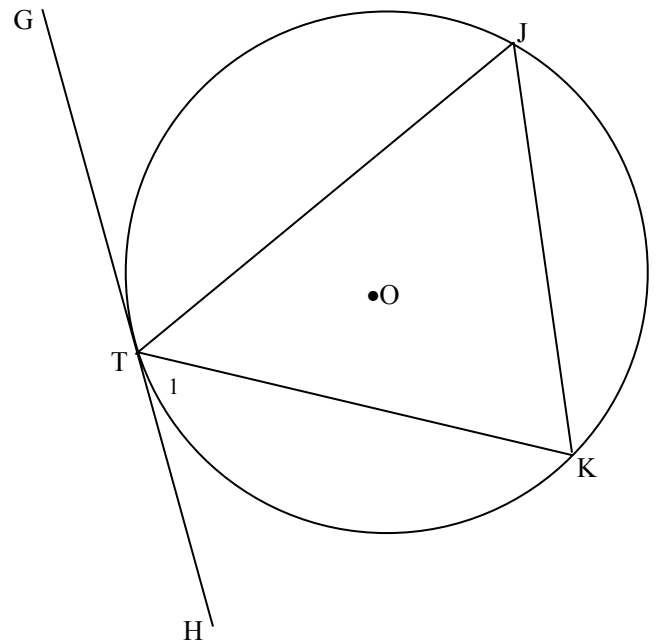
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DIAGRAM SHEET 2

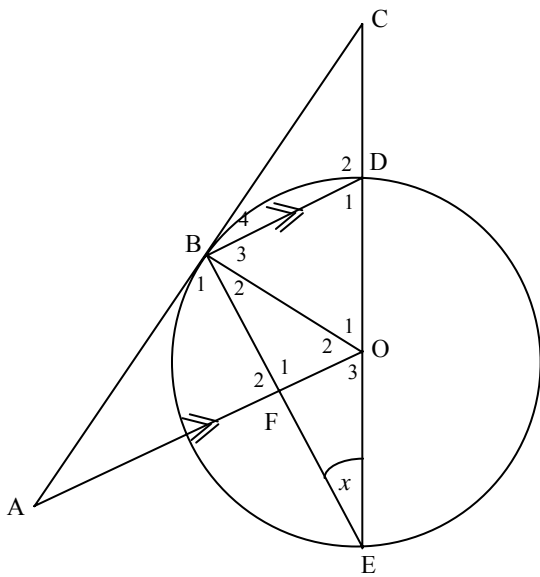
QUESTION 7



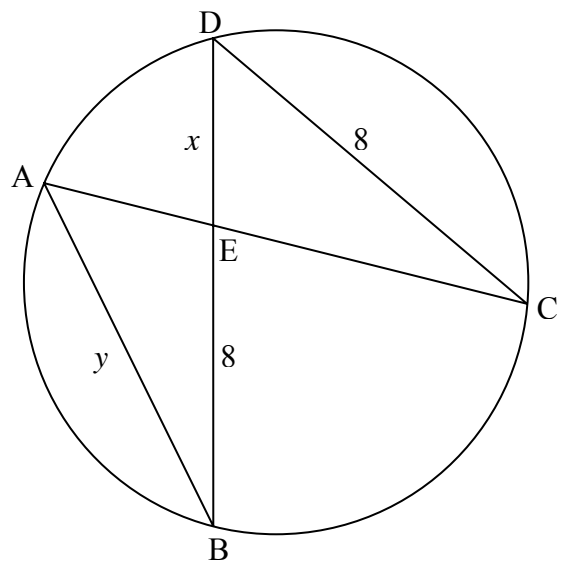
QUESTION 8.1



QUESTION 8.2



QUESTION 9



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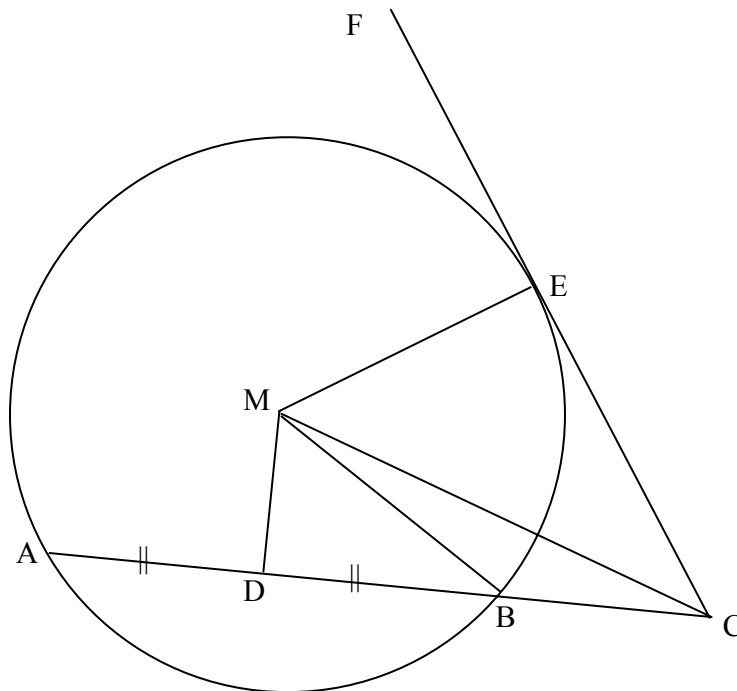
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DIAGRAM SHEET 3

QUESTION 10



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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GRADE 12

MATHEMATICS P3

NOVEMBER 2010

MEMORANDUM

MARKS: 100

This memorandum consists of 15 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.

QUESTION 1

<p>1.1</p>		<p>(7)</p>
<p>1.2</p>	<p> $5 + 12 + 2 + x + 75 - x + 66 - x + 3 + 2 = 103$ $x = 62$ </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Although CA applies to the question, the candidate cannot have negative or fraction answers.</p> </div>	<p> ✓ equation ✓ answer (2) </p>
<p>1.3.1</p>	<p> $P(\text{only eats chicken and fish and no vegetables}) = \frac{4}{103}$ </p>	<p> ✓ 4 ✓ 103 (2) </p>
<p>1.3.2</p>	<p> $P(\text{any two}) = \frac{12 + 4 + 13}{103} = \frac{29}{103}$ </p> <p>Accept</p> <p> $P(\text{any two}) = \frac{91}{103}$ </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: Although CA applies to the question, the candidate cannot have negative or value greater than 1.</p> </div>	<p> ✓ adding probabilities ✓ $\frac{29}{103}$ (2) </p> <p> ✓ adding probabilities ✓ $\frac{91}{103}$ </p>

[13]

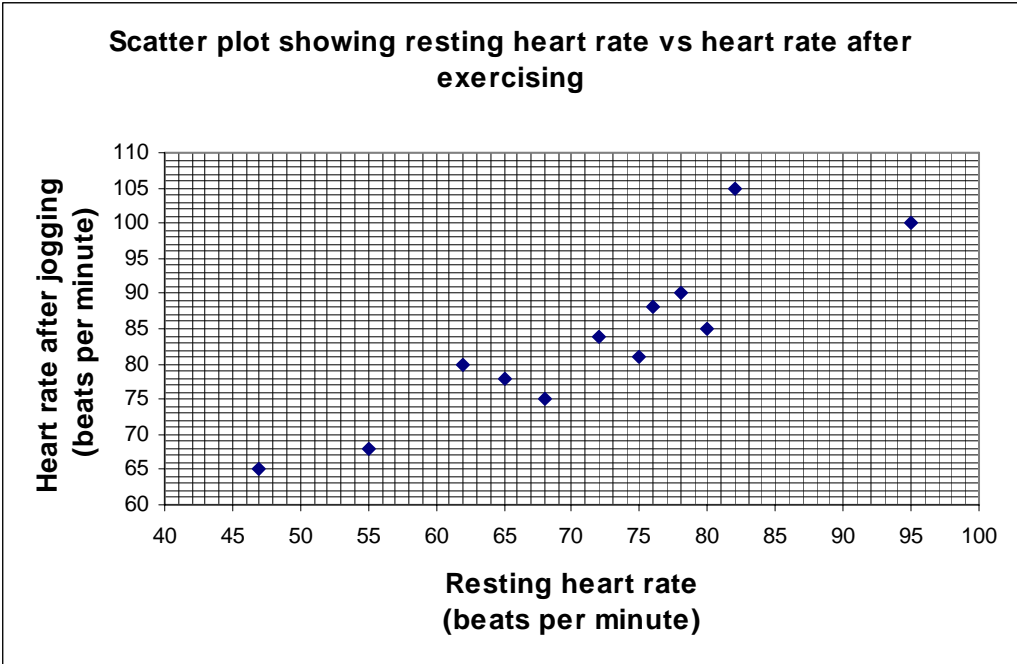
QUESTION 2

2.1	<p>No. They chose a Wednesday morning, when most people are at work. This is not a reliable time to do a survey about customer satisfaction. Most supermarkets are not busy at this time. Only 130 customers of a possible very large sample were interviewed. This is a very small number in comparison to the total number of customers that use a supermarket in a week.</p> <p>Accept: Yes, with a reasonable justification related to real life situations for example: very small rural community.</p> <p>Note: If the candidate answers YES or NO ONLY, then 0 / 2 marks.</p>	<p>No ✓✓ acceptable reason</p> <p>Yes ✓✓ acceptable reason</p> <p>(2)</p>
2.2	$\frac{22}{100} \times 130 = 28,6$ <p style="text-align: center;">OR</p> $\frac{78}{100} \times 130 = 101,4$ $130 - 101,4 = 28,6$ <p>Accept: 28 or 29</p>	<p>✓ $\frac{22}{100}$ or 22%</p> <p>✓ 28 or 29 or 28,6</p> <p>(2)</p>
2.3	<p>Choose a time when your store is busy, possibly Saturday or Sunday mornings. Interview more people to get a realistic point of view on customer service. Observe customer service over a longer period of time. Make use of questionnaires.</p> <p>Note: If yes in 2.1, the reasons must be relevant.</p>	<p>✓✓ any two valid reasons</p> <p>(2) [6]</p>

QUESTION 3

3.1	$\frac{68}{100} \times 20000$ $= 13\,600$ <p>OR</p> $\frac{66,7}{100} \times 20000$ $= 13\,340$ <p>OR</p> $\frac{68,3}{100} \times 20000$ $= 13\,660$	✓ 68 or 66,7 or 68,3 or $\frac{2}{3}$ ✓ answer (2)
3.2	<p>Lowest weight $= 182 - 3(0,454)$ $= 180,638$ grams Range $= 183,362 - 180,638$ $= 2,724$</p> <p>OR</p> <p>Range $= 6 \times 0,454$ $= 2,724$</p> <p>Accept:</p> <p>Range $= 8 \times 0,454$ $= 3,632$</p>	<p>Highest Weight $= 182 + 3(0,454)$ $= 183,362$ grams</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Answer only: full marks</p> <p>If candidate uses one or two standard deviations: max 2 marks</p> </div> <p>✓ correct 3 sd ✓ lowest weight ✓ highest weight ✓ difference (4)</p> <p>✓✓ 6 ✓ 0,454 ✓ answer (4) [6]</p>

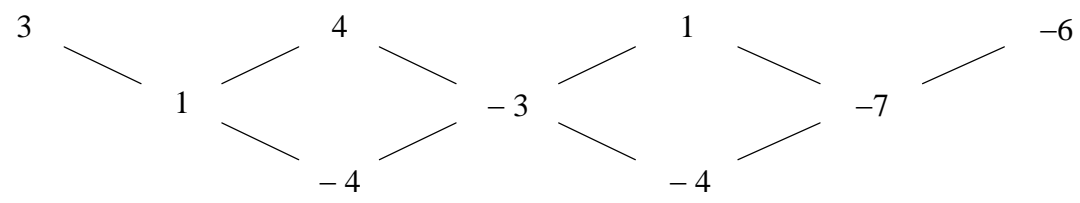
QUESTION 4

<p>4.1</p>	<p style="text-align: center;">Scatter plot showing resting heart rate vs heart rate after exercising</p>  <table border="1" style="display: none;"> <caption>Data points from the scatter plot</caption> <thead> <tr> <th>Resting heart rate (beats per minute)</th> <th>Heart rate after jogging (beats per minute)</th> </tr> </thead> <tbody> <tr><td>47</td><td>65</td></tr> <tr><td>55</td><td>68</td></tr> <tr><td>62</td><td>80</td></tr> <tr><td>65</td><td>78</td></tr> <tr><td>68</td><td>75</td></tr> <tr><td>72</td><td>84</td></tr> <tr><td>75</td><td>81</td></tr> <tr><td>78</td><td>88</td></tr> <tr><td>81</td><td>85</td></tr> <tr><td>83</td><td>90</td></tr> <tr><td>86</td><td>105</td></tr> <tr><td>95</td><td>100</td></tr> </tbody> </table>	Resting heart rate (beats per minute)	Heart rate after jogging (beats per minute)	47	65	55	68	62	80	65	78	68	75	72	84	75	81	78	88	81	85	83	90	86	105	95	100	<p>✓✓✓ all 12 points plotted correctly</p> <p>✓✓ 7 – 11 points plotted correctly</p> <p>✓ 2 – 6 points plotted correctly</p> <p style="text-align: right;">(3)</p>
Resting heart rate (beats per minute)	Heart rate after jogging (beats per minute)																											
47	65																											
55	68																											
62	80																											
65	78																											
68	75																											
72	84																											
75	81																											
78	88																											
81	85																											
83	90																											
86	105																											
95	100																											
<p>4.2</p>	<p> $a = 25,23$ (25,22587269...) $b = 0,81$ (0,8143737166...) $\hat{y} = a + bx$ $\hat{y} = 25,23 + 0,81x$ </p> <p>If using pen and paper method: $\bar{x} = 71,25$ $\bar{y} = 83,25$ $a = 25,23$ (25,22587269...) $b = 0,81$ (0,8143737166...) $\hat{y} = a + bx$ $\hat{y} = 25,23 + 0,81x$ </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If the line of best fit is drawn and its equation then calculated: 0 / 4 marks</p> </div>	<p>✓✓ a or b ✓ b or a</p> <p>✓ $\hat{y} = 25,23 + 0,81x$ (4)</p> <p>✓ \bar{x}, \bar{y} ✓ a ✓ b</p> <p>✓ $\hat{y} = 25,23 + 0,81x$ (4)</p>																										
<p>4.3</p>	<p> $r = 0,898$ $= 0,90$ (0,8979098935...) </p>	<p>✓✓ answer (2)</p>																										
<p>4.4</p>	<p>It is a very strong positive relationship.</p>	<p>✓ strong ✓ positive (2)</p>																										
<p>4.5</p>	<p> $\hat{y} = 25,23 + 0,81x$ $86 = 25,23 + 0,81x$ $x = 75,024...$ Resting heart rate could be 75 beats per minute. </p> <p>If a and b are not rounded off in the calculation, $x = 74,626 ...$ $x = 74,63$ </p> <p>If candidate draws in the least square regression line and reads of x-value where $y = 86$: full marks</p>	<p>✓ substitute $\hat{y} = 86$</p> <p>✓ answer (2)</p> <p>Accept $x = 74,63$ [13]</p>																										

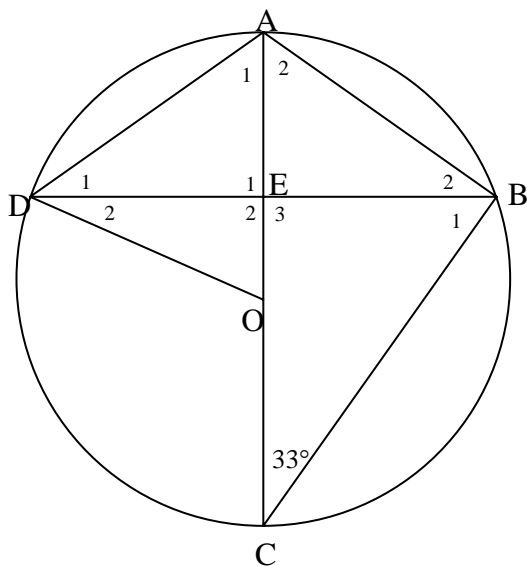
QUESTION 5

5.1	Number licence plates available $= 21 \times 21 \times 21 \times 10 \times 10 \times 10$ $= 21^3 \cdot 10^3$ $= 9\,261\,000$	✓ 21 ✓ 10 ✓ answer (3)
5.2	P(starting with Y) $= \frac{1 \times 21 \times 21 \times 10 \times 10 \times 10}{21 \times 21 \times 21 \times 10 \times 10 \times 10}$ $= \frac{441\,000}{9\,261\,000}$ $= \frac{1}{21}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: full marks</div>	✓ $21^2 \times 10^3$ ✓ denominator (CA with 5.1) ✓ answer (3)
5.3	P(contains number 7) $= \frac{21 \times 21 \times 21 \times 1 \times 9 \times 9 + 21 \times 21 \times 21 \times 9 \times 1 \times 9 + 21 \times 21 \times 21 \times 9 \times 9 \times 1}{9\,261\,000}$ $= \frac{3(21^3) \cdot 1.9.9}{9\,261\,000}$ $= \frac{243}{1000} \text{ or } 0,243$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">If did not multiply by 3: max 2</div> <p>OR</p> P(contains number 7) $= \frac{1 \times 9 \times 9 + 9 \times 1 \times 9 + 9 \times 9 \times 1}{1000}$ $= \frac{243}{1000} \text{ or } 0,243$	✓ 3 ✓ 1.9.9 ✓ denominator (3) ✓ 3 or $1 \times 9 \times 9 + 9 \times 1 \times 9 + 9 \times 9 \times 1$ ✓ 1.9.9 ✓ denominator
5.4	Number of unique number plates available with no repetition $= 21 \times 20 \times 19 \times 10 \times 9 \times 8$ $= 5\,745\,600$ <p>OR</p> ${}^{21}P_3 \cdot {}^{10}P_3$ $= \frac{21!}{18!} \times \frac{10!}{7!}$ $= 5\,745\,600$	✓ $21 \times 20 \times 19$ ✓ $10 \times 9 \times 8$ ✓ answer (3) ✓ ${}^{21}P_3$ ✓ ${}^{10}P_3$ ✓ answer (3) [12]

QUESTION 6

<p>6.1</p>	<p> $T_1 = 3$ $T_{1+1} = 3 - 4(1) + 5 = 4$ $T_{2+1} = 4 - 4(2) + 5 = 1$ $T_{3+1} = 1 - 4(3) + 5 = -6$ </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>If 3 ; 0 ; - 7 ; - 18 : max 2 marks</p> </div>	<p> ✓ T_2 ✓ T_3 ✓ T_4 (3) </p>
<p>6.2</p>	<p> Quadratic sequence. It adds a linear sequence to the preceding term. OR  <p> Quadratic Sequence Constant second difference of - 4 OR Recursive Need the previous term to calculate the next term </p> </p>	<p> ✓ quadratic ✓ reason (2) </p> <p> ✓ recursive ✓ reason (2) [5] </p>

QUESTION 7



<p>7.1</p>	<p>$\hat{D}_1 = 33^\circ$ (\angle in same segment) $\hat{A}\hat{E}D = 90^\circ$ (given) $\hat{A}_1 = 57^\circ$ (\angle sum Δ)</p> <p>OR $\hat{B}\hat{E}C = 90^\circ$ (given) $\hat{B}_1 = 57^\circ$ (\angle sum Δ) $\hat{A}_1 = 57^\circ$ (\angle in same segment)</p> <p>OR $DE = EB$ (line from circ cent \perp ch bis ch) AE is common $\hat{A}\hat{E}D = \hat{E}_1 = 90^\circ$ (given) $\Delta AED \equiv \Delta AEB$ (SAS) $\hat{A}\hat{B}C = 90^\circ$ (\angles in semi-circle) $\hat{A}_1 = \hat{A}_2 = 57^\circ$ (\angle sum Δ)</p>	<p>$\checkmark \hat{D}_1 = 33^\circ$ $\checkmark \angle$ in same segment $\checkmark \hat{A}_1 = 57^\circ$ (3)</p> <p>$\checkmark \hat{B}_1 = 57^\circ$ $\checkmark \hat{A}_1 = 57^\circ$ $\checkmark \angle$ in same segment (3)</p> <p>$\checkmark DE = EB$ (S/R)</p> <p>$\checkmark \Delta AED \equiv \Delta AEB$ (SAS)</p> <p>\checkmark answer (3)</p>
<p>7.2</p>	<p>$\hat{D}_2 + \hat{D}_1 = 57^\circ$ ($OD = OA =$ radii) $\hat{D}_2 = 24^\circ$</p> <p>OR $\hat{D}\hat{O}C = 114^\circ$ ($OD = OA =$ radii) OR \angle at the centre theorem $\hat{E}_2 = 90^\circ$ $\hat{D}_2 = 114^\circ - 90^\circ$ $= 24^\circ$</p>	<p>$\checkmark \hat{D}_2 + \hat{D}_1 = 57^\circ$ \checkmark answer (2)</p> <p>$\checkmark \hat{D}\hat{O}C = 114^\circ$</p> <p>$\checkmark$ answer (2)</p>

7.3	$\hat{A}BC = 90^\circ$ (\angle in semi-circle) $\hat{A}_2 = 57^\circ$ (\angle sum Δ) $= \hat{A}_1$ AE bisects $D\hat{A}B$ OR $DE = EB$ (line from circ centre bis ch) AE is common $\hat{E}_1 = \hat{A}EB = 90^\circ$ (given) $\Delta ADE \equiv \Delta ABE$ (SAS) $\hat{A}_2 = \hat{A}_1$	$\checkmark \hat{A}BC = 90^\circ$ $\checkmark \angle$ in semi-circle $\checkmark \hat{A}_2 = \hat{A}_1$ or AE bisects $D\hat{A}B$ (3) $\checkmark DE = EB$ (S/R) $\checkmark \Delta AED \equiv \Delta AEB$ (SAS) $\checkmark \hat{A}_2 = \hat{A}_1$ or AE bisects $D\hat{A}B$ (3) [8]
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QUESTION 8

8.1 Draw diameter TP.

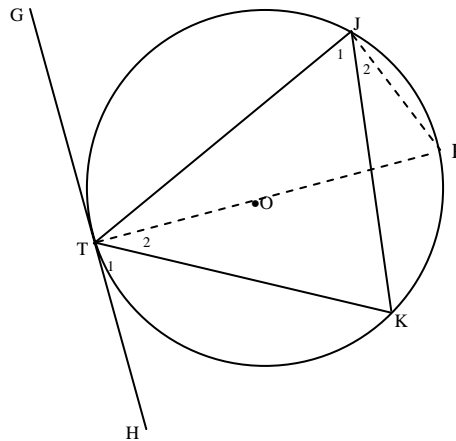
Join P to J.

$$\hat{T}_1 + \hat{T}_2 = 90^\circ \quad (\text{tan} \perp \text{diameter})$$

$$\hat{J}_1 + \hat{J}_2 = 90^\circ \quad (\angle \text{ in semi-circle})$$

$$\hat{J}_2 = \hat{T}_2 \quad (\angle \text{ in same seg})$$

$$T\hat{J}K = \hat{T}_1$$



✓ construction

✓ $\hat{T}_1 + \hat{T}_2 = 90^\circ$

✓ tan \perp diameter

✓ S/R

✓ S/R

(5)

OR

Draw radii OT and OK

Let $\hat{T}_2 = x$

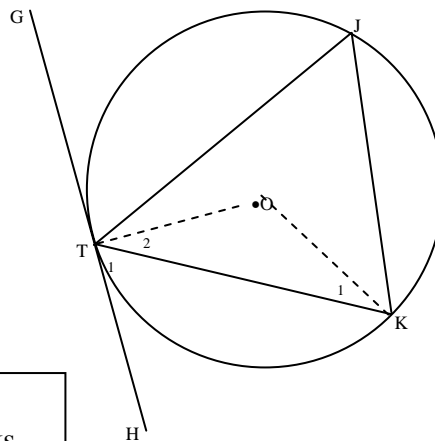
$$\hat{K}_1 = x \quad (\angle \text{ opp} = \text{ radii})$$

$$\hat{T}_1 = 90^\circ - x \quad (\text{rad} \perp \text{tan})$$

$$T\hat{O}K = 180^\circ - 2x \quad (\angle \text{ sum } \Delta)$$

$$T\hat{J}K = 90^\circ - x \quad (\angle \text{ circ cent})$$

$$T\hat{J}K = \hat{T}_1 \quad (= 90^\circ - x)$$



✓ construction

✓ $\hat{T}_1 = 90^\circ - x$

✓ rad \perp tan

✓ S/R

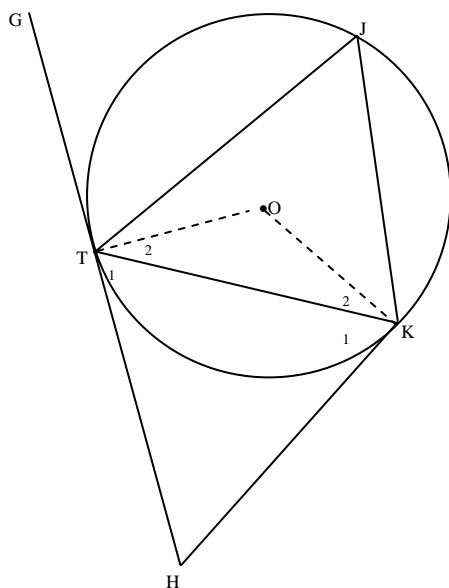
✓ S/R

(5)

NOTE:

If there is no construction: 0 / 5 marks

If candidate changes lettering and states "Similarly": max full marks



OR

Draw GT extend to H. Draw tangent KH at K.

$$TH = KH \quad (\text{tan from comm pt})$$

$$\hat{K}_1 = \hat{T}_1 \quad (\angle s \text{ opp} = \text{sides})$$

$$T\hat{O}K = 2T\hat{J}K$$

$$(\angle \text{ circ cent} = 2\angle \text{ circumf})$$

$$\hat{T}_1 + \hat{T}_2 = 90^\circ \quad (\text{tan} \perp \text{radius})$$

$$T\hat{O}K = 180^\circ - (90^\circ - \hat{T}_1 + 90^\circ - \hat{K}_1)$$

$$= \hat{T}_1 + \hat{K}_1$$

$$= \hat{T}_1 + \hat{T}_1$$

$$= 2\hat{T}_1$$

$$\hat{T}_1 = \frac{1}{2}K\hat{O}T$$

$$= T\hat{J}K$$

✓ construction

✓ S/R

✓ S/R

✓ $\hat{T}_1 + \hat{T}_2 = 90^\circ$

✓ tan \perp radius

(5)

OR

Construct OT, OJ and OK

$$\hat{T}_1 = \hat{J}_1 = x \quad (\text{radii})$$

$$\hat{T}_2 = \hat{K}_1 = z \quad (\text{radii})$$

$$\hat{K}_2 = \hat{J}_2 = y \quad (\text{radii})$$

$$2x + 2y + 2z = 180^\circ \quad (\angle \text{ sum } \Delta)$$

$$x + y + z = 90^\circ$$

$$x + y = 90^\circ - z$$

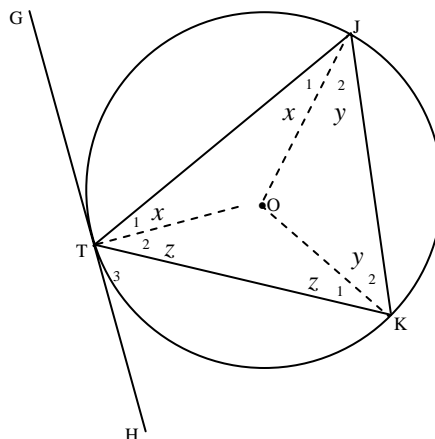
$$O\hat{T}H = 90^\circ \quad (\text{rad } \perp \text{ tan})$$

$$\hat{T}_3 = 90^\circ - z$$

$$= 90^\circ - (90^\circ - (x + y))$$

$$= 90^\circ - z$$

$$= T\hat{J}K$$



✓ construction
✓ S/R

✓ S

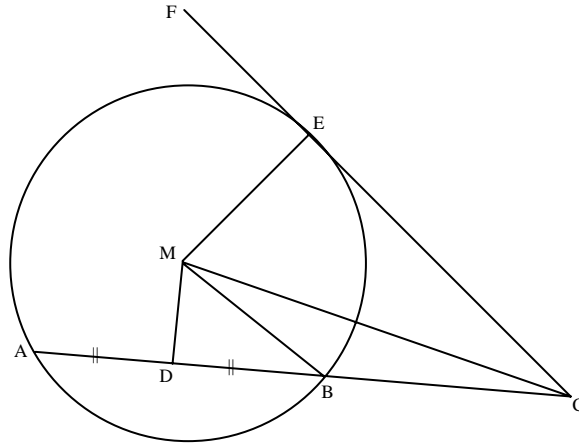
✓
 $\hat{T}_3 + \hat{T}_2 = 90^\circ$
✓ rad \perp tan

(5)

<p>8.2</p>		
<p>8.2.1</p>	<p>$\hat{B}_4 = x$ (tan chord theorem)</p> <p>$\hat{A} = \hat{B}_4 = x$ (corres \angle; $BD \parallel AO$)</p> <p>$\hat{B}_2 = x$ ($BO = EO =$ radii)</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Note: If start with $\hat{A} = x$ and do not use tan ch th: max 2 marks</p> </div>	<p>✓ $\hat{B}_4 = x$</p> <p>✓ tan chord theorem</p> <p>✓ $\hat{A} = \hat{B}_4 = x$ with reason</p> <p>✓ $\hat{B}_2 = x$</p> <p style="text-align: right;">(4)</p>
<p>8.2.2</p>	<p>$\hat{D}\hat{B}E = 90^\circ$ (\angle in semi-circle)</p> <p>$\hat{C}\hat{B}E = 90^\circ + x$</p> <p>OR</p> <p>$\hat{C}\hat{B}O = 90^\circ$ (rad \perp tan)</p> <p>$\hat{C}\hat{B}E = 90^\circ + x$</p> <p>OR</p> <p>$\hat{O}_1 = 2x$ (\angle circ cent)</p> <p>$\hat{B}_3 = \hat{D}_1 = 90^\circ - x$ (radii)</p> <p>$\hat{C}\hat{B}E = x + (90^\circ - x) + x = 90^\circ + x$</p>	<p>✓ $\hat{D}\hat{B}E = 90^\circ$</p> <p>✓ \angle in semi-circle</p> <p>✓ $\hat{C}\hat{B}E = 90^\circ + x$</p> <p style="text-align: right;">(3)</p> <p>✓ $\hat{C}\hat{B}O = 90^\circ$</p> <p>✓ rad \perp tan</p> <p>✓ $\hat{C}\hat{B}E = 90^\circ + x$</p> <p style="text-align: right;">(3)</p> <p>✓ $\hat{O}_1 = 2x$</p> <p>✓ \angle circ cent</p> <p>✓ $\hat{C}\hat{B}E = 90^\circ + x$</p> <p style="text-align: right;">(3)</p>
<p>8.2.3</p>	<p>$\hat{D}\hat{B}E = 90^\circ$ (proved in 8.2.2)</p> <p>$\hat{B}\hat{F}O = 90^\circ$ (co-int angles supp; $BD \parallel AO$)</p> <p>$BF = FE$ (line from circ cent \perp ch bisect ch)</p> <p>F is the midpoint of EB</p>	<p>✓ $\hat{D}\hat{B}E = 90^\circ$</p> <p>✓ $\hat{B}\hat{F}O = 90^\circ$ and reason</p> <p>✓ $BF = FE$</p> <p>✓ line from circ cent \perp ch bisect ch</p> <p style="text-align: right;">(4)</p>

	<p>OR $OD = OE$ (radii) $BF = FE$ ($BD \parallel AO$) F is the midpoint of EB</p> <p>OR $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) OF is common $BO = OE$ (radii) $\triangle BOF \equiv \triangle EOF$ (90°HS) $BF = FE$ ($\equiv \triangle$s)</p> <p>OR $\hat{B}_2 = \hat{A} = x$ (proven) \hat{O}_2 is common $\triangle AOB \parallel \triangle BOF$ (AAA) $\hat{AO} = \hat{BFO}$ $\hat{AO} = 90^\circ$ (proven) $\hat{AO} = \hat{BFO} = 90^\circ$ $BF = FE$ (line from circ cent \perp ch bisects ch)</p> <p>OR $\hat{DBE} = 90^\circ$ (\angle in semi-circle) $\hat{B}_3 = 90^\circ - x$ $\hat{O}_2 = 90^\circ - x$ (alt \angles; $BD \parallel FO$) $\hat{F}_1 = 90^\circ$ (\angle sum \triangle) $BF = FE$ (line from circ cent \perp ch bisects ch)</p> <p>OR In $\triangle OBF$ and $\triangle OEF$ 1. $OB = OE$ (radii) 2. $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) 3. $\hat{B}_2 = \hat{E}$ (radii) $\triangle OBF \equiv \triangle OEF$ (AAS) $BF = FE$</p>	<p>✓ $OD = OE$ ✓ radii ✓ $BF = FE$ ✓ $BD \parallel AO$ (4)</p> <p>✓ $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) ✓ $BO = OE$ ✓ $\triangle BOF \equiv \triangle EOF$ ✓ $BF = FE$ (4)</p> <p>✓ $\triangle AOB \parallel \triangle BOF$ ✓ $\hat{AO} = \hat{BFO}$ ✓ $BF = FE$ ✓ line from circ cent \perp ch bisects ch (4)</p> <p>✓ $\hat{DBE} = 90^\circ$ ✓ $\hat{F}_1 = 90^\circ$ ✓ $BF = FE$ ✓ line from circ cent \perp ch bisects ch (4)</p> <p>✓ $OB = OE$ ✓ $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) ✓ $\triangle OBF \equiv \triangle OEF$ ✓ $BF = FE$ (4)</p>
<p>8.2.4</p>	<p>In $\triangle CBD$ and $\triangle CEB$ 1. $\hat{E} = \hat{B}_4 = x$ (proven in 8.2.1) 2. \hat{C} is common 3. $\hat{D}_4 = \hat{CBE} = 90^\circ + x$ $\triangle CBD \parallel \triangle CEB$ (AAA)</p>	<p>✓ $\hat{E} = \hat{B}_4 = x$ ✓ \hat{C} is common Or ✓ $\hat{D}_4 = \hat{CBE} = 90^\circ + x$ Any two of the above (2)</p>

QUESTION 10



<p>10.1</p>	<p> $\widehat{MEC} = 90^\circ$ (tan \perp rad) $\widehat{MDC} = 90^\circ$ (line from cent bisects ch) $\widehat{MEC} + \widehat{MDC} = 180^\circ$ \therefore MDCE a cyclic quad (opp \angles of quad supplementary) </p> <p>OR</p> <p> $\widehat{MEC} = 90^\circ$ (tan \perp rad) $\widehat{MDA} = 90^\circ$ (line from cent bisects ch) $\widehat{MEC} = \widehat{MDA}$ \therefore MDCE a cyclic quad (ext \angle quad = int opp) </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>NOTE: If the word <i>cyclic</i> is used in the last reason: max 2 / 3 marks</p> </div>	<p> ✓ $\widehat{MEC} = 90^\circ$ (tan \perp rad) ✓ $\widehat{MDC} = 90^\circ$ ✓ opp \angles of quad supplementary (3) ✓ $\widehat{MEC} = 90^\circ$ (tan \perp rad) ✓ $\widehat{MDA} = 90^\circ$ ✓ ext \angle quad = int opp (3) </p>
<p>10.2</p>	<p> $MD^2 = MB^2 - DB^2$ (Pythagoras; $\triangle MBD$) $MC^2 = MD^2 + DC^2$ (Pythagoras; $\triangle MDC$) $= MB^2 - DB^2 + DC^2$ </p>	<p> ✓ $MD^2 = MB^2 - DB^2$ ✓ Pythagoras ✓ $MC^2 = MD^2 + DC^2$ (3) </p>
<p>10.3</p>	<p> $DB = 30$ (given) $MB = 40$ (radii) $MC^2 = (40)^2 + (50)^2 - (30)^2$ $= 3\ 200$ $MC = 40\sqrt{2} = 56,57$ $MC^2 = ME^2 + CE^2$ (Pythagoras) $CE^2 = 3\ 200 - 1\ 600$ $CE^2 = 1\ 600$ $CE = 40$ mm </p> <p>OR</p> <p> $MC^2 = CE^2 + ME^2 - 2CE \cdot ME \cdot \cos \widehat{MEC}$ $3200 = CE^2 + (40)^2 - 2CE \cdot (40) \cdot \cos 90^\circ$ $= CE^2 + 1600$ $CE^2 = 1600$ $CE = 40$ </p>	<p> ✓ $MB = ME$ ✓ $DB = 30$ ✓ $MC^2 = 3200$ or $MC = 40\sqrt{2}$ or $MC = 56,57$ </p> <p> ✓ answer (4) </p> <p> ✓ cosine rule ✓ $ME = 40$ ✓ $MC^2 = 3200$ </p> <p> ✓ answer (4) </p> <p style="text-align: right;">[10]</p>